

# Engineering Guide to Spring Design



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# CONTENTS

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ABBREVIATIONS AND SYMBOLS .....	9
GENERAL NOMENCLATURE.....	10
PREFACE .....	11
HOW TO USE THE GUIDE .....	12
SELECTING SPRING CONFIGURATIONS.....	14
1 SPRING MATERIALS .....	21
1.1 Chemical and Physical Characteristics.....	21
1.2 Elastic Modulus.....	23
1.3 Magnetic Characteristics.....	23
1.4 Heat Treatment of Springs.....	23
1.5 Environmental Considerations .....	26
1.6 Stress Relaxation .....	26
1.7 Corrosion .....	28
1.8 Spring Wire.....	30
1.9 Cost and Availability .....	30
1.10 Spring Strip.....	31
1.11 Other Spring Materials .....	34
1.12 Specifying Hardness.....	34
2 RESIDUAL STRESS, FATIGUE, AND RELIABILITY.....	38
2.1 Introduction.....	38
2.2 Residual Stresses.....	38
2.3 Residual Stresses for Increased Load-Carrying Ability .....	38
2.4 Residual Stresses for Increased Fatigue Strength .....	39
2.5 Fatigue.....	40
2.6 Fatigue Life Prediction .....	41
2.7 Reliability .....	41
2.8 Reliability and Fatigue .....	42
2.9 Reliability and Load Loss.....	43
3 HELICAL COMPRESSION SPRINGS .....	44
3.1 Introduction.....	44
3.2 Helical Compression Spring Terminology.....	44

3.3	Spring Diameter.....	44
3.4	Spring Index.....	45
3.5	Free Length.....	45
3.6	Type of Ends.....	45
3.7	Number of Coils.....	45
3.8	Solid Height.....	45
3.9	Direction of Coiling.....	47
3.10	Squareness and Parallelism.....	47
3.11	Hysteresis.....	47
3.12	Spring Rate.....	47
3.13	Stress.....	48
3.14	Loads.....	48
3.15	Buckling of Compression Springs.....	49
3.16	Choice of Operating Stress—Static Conditions.....	49
3.17	Choice of Operating Stress—Cyclic Applications.....	51
3.18	Fatigue Life Estimation Example.....	52
3.19	Dynamic Loading—Impact.....	53
3.20	Dynamic Loading—Resonance.....	55
3.21	Special Springs.....	56
3.22	Commercial Tolerances.....	60
3.23	Acceptable Quality Level (AQL).....	60
3.24	Packaging.....	60
3.25	How to Specify.....	61
3.26	Compression Spring Design Example.....	61
4	HELICAL EXTENSION SPRINGS.....	64
4.1	Introduction.....	64
4.2	Initial Tension.....	64
4.3	Types of Ends.....	65
4.4	Extension Spring Dimensions.....	66
4.5	Design Equations.....	66
4.6	Choice of Operating Stress—Static.....	67
4.7	Choice of Operating Stress—Cyclic.....	67
4.8	Clearances.....	67

4.9	Tolerances.....	67
4.10	How to Specify .....	68
4.11	Extension Spring Design Example .....	70
4.12	Final Specifications .....	71
5	GARTER SPRINGS .....	72
5.1	Introduction.....	72
5.2	Joint Design and Considerations .....	72
5.3	Spring Rate.....	72
5.4	Radial and Circumferential Load .....	72
5.5	Initial Tension and Stress.....	73
5.6	Stress .....	73
5.7	How to Specify.....	73
5.8	Garter Spring Design Example .....	74
5.9	Final Design Specifications .....	75
6	HELICAL TORSION SPRINGS.....	76
6.1	Introduction.....	76
6.2	Number of Turns .....	76
6.3	Mean Diameter .....	76
6.4	Length.....	77
6.5	Spring Rate.....	77
6.6	Stress .....	77
6.7	End Configurations .....	78
6.8	Choice of Operating Stress—Static.....	78
6.9	Choice of Operating Stress—Cyclic .....	79
6.10	Double Torsion Springs.....	79
6.11	Rectangular Wire .....	79
6.12	Tolerances .....	80
6.13	How to Specify .....	80
6.14	Design Example .....	80
6.15	Final Design Specifications.....	81
7	RETAINING RINGS.....	83
7.1	Introduction.....	83
7.2	End Configurations .....	83

7.3	Loads and Deflection .....	83
7.4	Stress .....	84
7.5	Choice of Stress Level.....	84
7.6	Tolerances.....	84
7.7	How to Specify.....	84
7.8	Retaining Ring Design Example.....	85
7.9	Final Design Specifications .....	85
8	BELLEVILLE SPRING WASHERS.....	87
8.1	Introduction.....	87
8.2	Design Considerations.....	87
8.3	Design Calculations .....	88
8.4	Choice of Stress Level—Static.....	92
8.5	Choice of Stress Level—Cyclic .....	93
8.6	Stacks of Belleville Washers.....	93
8.7	Tolerances.....	95
8.8	How to Specify.....	95
8.9	Belleville Spring Washer Design Example.....	95
8.10	Final Design Specifications.....	96
9	FLAT SPRINGS .....	98
9.1	Introduction.....	98
9.2	Design Considerations.....	98
9.3	Design Equations.....	98
9.4	Cantilever Springs .....	99
9.5	Simple Beam or Elliptical Springs .....	99
9.6	Choice of Operating Stress.....	100
9.7	Tolerances.....	100
9.8	How to Specify.....	100
9.9	Flat Spring Design Example .....	101
9.10	Final Design Specifications.....	101
10	SPECIAL SPRING WASHERS.....	103
10.1	Introduction .....	103
10.2	Curved Washers .....	103
10.3	Wave Washers .....	103

10.4	Finger Washers .....	104
10.5	Choice of Operating Stress—Static .....	104
10.6	Choice of Operating Stress—Cyclic.....	104
10.7	Tolerances .....	104
10.8	How to Specify .....	105
10.9	Special Spring Washer Design Example.....	105
10.10	Final Design Specifications .....	106
11	POWER SPRINGS .....	107
11.1	Introduction .....	107
11.2	Design Considerations .....	107
11.3	Design Equations .....	108
11.4	How to Specify .....	110
11.5	Power Spring Design Example .....	111
11.6	Final Design Specifications.....	111
12	CONSTANT FORCE SPRINGS.....	113
12.1	Introduction .....	113
12.2	Extension Type .....	113
12.3	Tolerances .....	114
12.4	Motor Type.....	114
12.5	Choice of Stress Levels .....	115
12.6	Tolerances .....	115
12.7	How to Specify .....	116
13	SPIRAL SPRINGS .....	117
13.1	Introduction .....	117
13.2	Hair Springs.....	117
13.3	Choice of Stress Level .....	118
13.4	Tolerances .....	118
13.5	Brush Springs.....	118
13.6	Choice of Stress Level .....	118
13.7	Tolerances .....	118
13.8	How to Specify .....	119
13.9	Brush Spring Design Example .....	119
14	VOLUTE SPRINGS .....	120

14.1	Introduction .....	120
14.2	Choice of Stress Level .....	120
14.3	Design Equations .....	120
15	WIRE FORMS .....	122
15.1	Introduction .....	122
15.2	How to Specify .....	122
	BIBLIOGRAPHICAL REFERENCES.....	123
	TRADEMARKS.....	123
	GLOSSARY OF SPRING TERMINOLOGY .....	125
	CONVERSION FACTORS .....	128
	CONVERSIONS FOR CARBON STEEL .....	129

## ABBREVIATIONS AND SYMBOLS

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Abbreviations		
HRC	hardness, Rockwell C scale	
Hz	hertz	
in.	inch	
J	joule	
kg	kilogram	
lbf	pound force	
m	meter	
mm	millimeter	
N	newton	
ozf	ounce force	
Pa	pascal	

Symbols		
M	mega	1,000,000
k	kilo	1,000
h	hecto	100
c	centi	0.01
m	milli	0.001
μ	micro	0.000001

## GENERAL NOMENCLATURE

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### General Nomenclature

A = Area, mm<sup>2</sup> (in<sup>2</sup>)

b = Width, mm (in)

C = Spring index, D/d

D = Mean diameter, (O.D.+I.D.)/2, mm (in)

I.D. = Inside diameter, mm (in)

O.D. = Outside diameter, mm (in)

d = Wire diameter, mm (in)

E = Modulus of elasticity in tension or Young's Modulus, MPa (psi)

f = Deflection, mm (in)

g = Gravitational constant, 9.807m/sec<sup>2</sup> (386.4 in/sec<sup>2</sup>)

G = Shear modulus or modulus of rigidity, MPa (psi)

I = Moment of inertia, mm<sup>4</sup> (in<sup>4</sup>)

K = Design constant

K<sub>w</sub> = Stress correction factor for helical springs

k = Spring rate, N/mm (lbf/in) or N-mm/revolution (lbf-in/revolution)

L = Length, mm (in)

L<sub>f</sub> = Free length, mm (in)

L<sub>s</sub> = Length at solid, mm (in)

M = Moment or torque, N-mm (lbf-in)

N<sub>a</sub> = Number of active coils or waves

N<sub>t</sub> = Total number of coils

n = Frequency, hertz

ρ = Density, g/cm<sup>3</sup> (lb/in<sup>3</sup>)

S = Stress, MPa (psi)

θ = Angular deflection, expressed in number of revolutions

TS = Tensile strength, MPa (psi)

t = thickness, mm (in)

YS = Yield strength, MPa (psi)

μ = Poisson's ratio

## PREFACE

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When the first spring was produced is not known; however, it was very likely a bow. The bow was used to store energy that could be released at a precise moment upon command by the bowman. The first formal study of flexible members was made by Robert Hooke in 1678. Hooke's famous law that "deflection is proportional to load" is still the basis for spring design. Owing to advances in stress measurement and prediction, material technology, and manufacturing methods, our knowledge of springs and springmaking has advanced considerably since Hooke's time.

Initially, springmakers were wire benders—people who shaped wire into a helix or other suitable form. As technology grew, manufacturers started to specialize. Springs must apply a force, or store and release a predictable amount of energy, over a relatively large deflection. Springs are more similar to an instrument than to a standard metal part. The wire benders and stampers who recognized this and became skilled in the art of spring design soon became members of a unique group of metal formers known as springmakers.

Many springmakers and designers today are increasingly specialized, choosing to focus on very specific spring types. In contrast, Associated Spring produces a large breadth of spring types backed by a global supply chain across their production facilities throughout the world.

This handbook is written for those who design parts to perform a spring function. The first section discusses different spring functions and relates them to specific configurations.

After reading the section titled "Selecting Spring Configurations," a designer can select one or possibly two spring configurations that will perform the desired function on a cost-effective

basis. The section titled "Spring Materials" is written in a similar manner to lead a designer to the most cost-effective material for an intended application. At some point in the design process, the designer must select the level of operating stress, weighing the benefits associated with improved reliability or performance against added cost. The section titled "Residual Stress, Fatigue, and Reliability" is provided as background information to assist the designer in making these judgments. Remaining sections show the designer how to design and specify various types of springs for many applications. Spring design is an interactive process, and may require repeated operations until the best design is achieved.

Because of spring complexity, many simplifying assumptions are made in the design process. Over the years, these assumptions have proven to be reliable. But because such assumptions exist, the calculations are not always exact. Therefore, for critical or unusual springs, these assumptions may not be satisfactory, and more complex design procedures may be required. In these situations, designers should rely on the experience of a springmaker who can use computer aided design, software design tools, finite element stress analysis, mechanical simulation, or sample parts to confirm the right specifications.

## HOW TO USE THE GUIDE

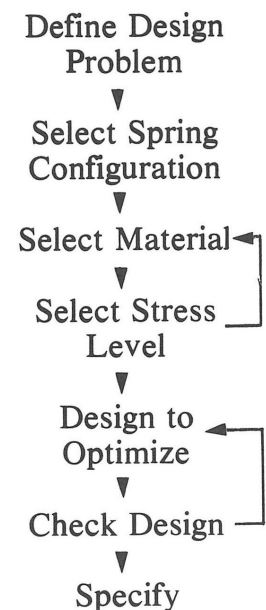
This handbook is written for design engineers as a practical guide for those responsible for designing springs. Springs are flexible members that store energy. Design considerations for members that experience large deflections are quite different from those used for rigid structures. A sufficient amount of detail has been included so that those designers without access to a computerized program can generate an optimal solution to a spring design problem.

Many designs do not perform well in service due to incomplete or unclear definition. The assumption is made throughout this handbook that the designer knows the functional requirements of a spring, its space limitations, the environment in which it operates, its service requirements, and any special considerations. Functional requirements are usually expressed as a load at a test position and/or a spring rate. Space limitations are defined by describing the envelope in which a spring is expected to operate. Environment can be characterized by the operating temperature and a description of substances in contact with a spring. Service requirements are the expected life, frequency of loading, rate of loading, and permissible relaxation. Special considerations might involve, for example, restrictions due to assembly, electrical conductivity, or magnetic requirements. To make a cost-effective design, it is essential to have the design problem clearly defined.

This handbook follows the design sequence illustrated in Figure 1-1. Section 2, "Selecting Spring Configuration," reviews the methodology for choosing the best type of spring configuration to perform an intended function. Frequently, the choice of configuration is obvious to experienced designers. The inexperienced designer—and occasionally the experienced designer (in critical situations)—should first review Section 2 to be

sure that the most cost-effective configuration has been selected. The second major design decision is choice of material. Section 3, "Spring Materials," gives information required to select one or two candidate materials for the design. Section 4, "Residual Stress, Fatigue, and Reliability," gives background information necessary to select a stress level that will yield a satisfactory balance between cost and reliability. Remaining sections discuss each type of spring configuration in detail. These sections enable a designer, having chosen the configuration and material, to select a stress level, design a spring, and then specify the spring to a springmaker. In many cases, examples are included to demonstrate the design process. In most sections, recommendations are made on stress levels for both fatigue and static service.

*Fig. 1-1. Design Sequence.*



Material recommendations and design methods discussed here are the result of many years of experience and have proven to be reliable. Spring design is a very complex subject. Frequently the state of stress is not accurately known. Geometrical configurations are often difficult to describe mathematically.

Simplifying assumptions have been made; in some cases, these assumptions may lead to inaccuracies. When parts are made to a design, fabrication considerations are occasionally encountered that alter performance.

Materials do not always behave as predicted. Although the best judgment has been used in writing this handbook, resulting designs will not always be optimal. For critical springs, it is advisable to call on the experience of Associated Spring's engineering staff, and to have samples made and tested prior to committing a design to production.

Notes on how to specify a spring, manufacturing tips, and other information dedicated to a specific spring configuration should be reviewed. This information is not generally available in design programs and yet frequently is necessary to achieve the most cost-effective design.

For many spring applications it is possible to use standard spring designs that save engineering time and avoid tooling costs and special manufacturing lot charges. Associated Spring has available more than 5,000 pre-engineered spring designs. Compression, extension, torsion, and constant force springs are maintained in stock, as are Belleville, curved, finger, and wave spring washers.

## SELECTING SPRING CONFIGURATIONS

It is essential that a designer select the optimum spring configuration to perform an intended function. A three-step procedure has been developed for this task:

1. Define the primary spring function in terms of push, pull, twist, or energy storage.
2. Review various alternative configurations and select one or two best candidates.
3. Review candidate configurations with respect to cost and special considerations.

Having defined primary spring function as either push, pull, or twist, the next step is to review all possible spring configurations that perform this function and then select the one that meets space requirements most economically. Various spring configurations are listed in Table 2-1 with helpful comments to aid in selecting one or two candidates. Helical compression springs, spring washers, volute springs, and beam springs all perform a push function. For large deflections, helical compression springs are most commonly chosen; for small deflections, spring washers are most common. Volute springs have high damping capacity and good resistance to buckling, but are not common because of relatively high manufacturing costs. Beam springs are produced in a wide variety of shapes and can push or pull. Frequently, beam springs are required for functions in addition to the spring function, and they are sometimes an integral element of a larger part.

Helical torsion and spiral spring configurations perform the twist function. Helical torsion springs are often used as a counterbalance for doors, lids, or other mechanisms that rotate on a shaft. Spiral hair springs have a low hysteresis and are used in instruments and watches.

Brush springs received their name from their primary application of holding brushes against commutators in electric motors. Power springs are often called clock or motor springs and are used to store energy for clocks, toys, and other devices.

A pre-stressed power spring is a special type of power spring with a very high Energy Storage Capacity (ESC) and is most commonly used on retractors for seat belts. Lev'l Torq® springs provide an essentially constant torque over many revolutions.

The pull function is performed by extension springs, drawbar spring assemblies, and constant force springs, with helical extension springs being most common. Drawbar spring assemblies are useful when a fixed stop is required. Constant force springs are similar to power springs; however, they are loaded by pull rather than twist.

Retaining rings and garter springs were especially developed to perform either push or pull. Retaining rings retain or locate parts in bearings and on shafts. Garter springs are used primarily in oil seals.

Often a spring design function is expressed in terms of ESC. In machines, springs are frequently used to store kinetic energy from moving components during deceleration; this energy is then released during acceleration to reduce peak loads. Spring motors are used to power clocks, toys, and many other mechanical devices. Torsion springs are used in window shades and garage doors, primarily for their ability to store energy.

ESC is defined as the amount of work done by a spring or the energy stored per unit volume of active spring material. ESC is proportional to the

square of the maximum operating stress level, divided by the modulus of elasticity, multiplied by a constant.








Theoretical capacities are shown for various spring configurations in Table 2-2. Space efficiency, another measure of spring design efficiency, is the volume of active spring material divided by the volume of the envelope occupied by the spring when fully deflected. The product of ESC and space efficiency is the amount of energy a spring configuration can store per unit volume of the envelope it occupies. Typical ranges of ESC per unit volume of envelope for some spring geometries are also listed in Table 2-2. These values are approximate, and refer to springs in fully deflected positions without regard to inactive material or stress correction factors. The space efficiency concept is not meaningful for some spring configurations, such as cantilevers and extension springs. Pre-stressed power springs, power springs, and helical compression spring designs are most suitable for energy storage applications.

ESC divided by the product of the density and cost per pound gives energy storage ability per unit cost. This is a convenient method for making rough comparisons of various spring materials.

The final step in the selection process is to consider other restrictions imposed by design criteria. Cost is always a restriction. Although specific comments on cost cannot be addressed until a spring is designed, some useful generalizations can be considered here. For sample and small quantities, Stock Precision Engineered Components (SPEC) are available from Associated Spring at considerably less cost than custom designed springs. Custom designed springs from wire are generally less costly than springs from strip. This is because there is very little scrap from wire. Many flat springs are blanked from strip with a concomitant loss

of material. Springs made from pre-hardened material tend to be less costly than springs hardened after forming. Sharp bends tend to increase manufacturing costs and cause stress concentrations that can result in early failure.

Compression springs and several other types of common springs are made on universal tooling. Most flat springs and special wire forms require special tooling. The ability to maintain tolerances varies considerably for different spring configurations. In general, tolerances controlled by metal forming processes are substantially greater than tolerances controlled by metal cutting.

TYPE	CONFIGURATION	ACTION
<b>Helical Compression (section 5)</b>		
Round and Rectangular Wire		
Constant Pitch		Push—wide load and deflection range—constant rate.
Conical		Push—wide load and deflection range. Conical spring can be made with minimum solid height and with constant or increasing rate. Barrel, hourglass, and variable-pitch springs used to minimize resonant surging and vibration.
Barrel		
Hourglass		
Variable Pitch		
<b>Spring Washer (Section 11 and 13)</b>		
Belleville		Push—high loads, low deflections—choice of rates (constant, increasing, or decreasing).
Wave		Push—light loads, low deflections—uses limited radial space.

Slotted



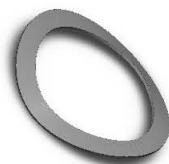
Push—higher deflections than Bellevilles.

Finger



Push—for axial loading of bearings.

Curved



Push—used to absorb axial end play.

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**Volute (Section 17)**

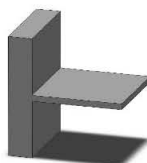


Push—may have an inherently high friction damping.

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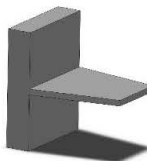
**Beam (Section 12)**

Cantilever, Regular Section



Push or pull—wide range of loads, low deflection range.

Cantilever, Trapezoidal Section



Simple Beam



**Helical Torsion (Section 9)**

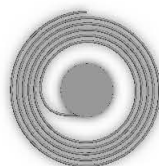
Round or Rectangular Wire



Twist—constant rate.

**Spiral (Section 16)**

Hairspring



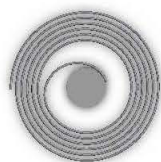
Twist

Brush



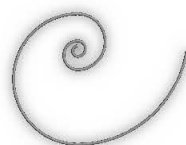
Twist or push

**Power, Motor or Clock (Section 14)**



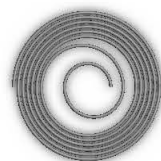
Twist—exerts torque over many turns.

Supplied in retainer.



Removed from retainer.

**Prestressed Power (Section 14)**



Twist—exerts torque over many turns.

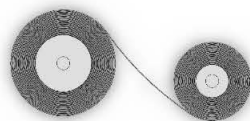
Supplied in retainer.



Removed from retainer.

**Constant (Section 15)  
Force Spring Motor**

Level Torq



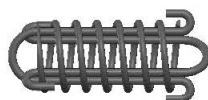
Twist—exerts close-to-constant torque over many turns.

**Helical Extension (Section 7)**



Pull—wide load and deflection range—constant rate.

**Drawbar (Section 7)**



Pull—extension to a solid stop.

**Constant Force (Section 15)**



Pull—very long deflection at constant load or low rate.

**Retaining Rings (Section 10)**

Round or Rectangular Wire



Pull or push to resist axial loads.

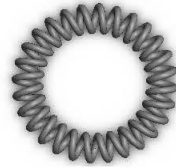
**Garter (Section 8)**

Extension



Pull with radial pressure.

Compression



Push with radial pressure.

**Table 2-2. Energy Storage Capacity (ESC) of Various Spring Configurations.**

*Table 2-2. Energy Storage Capacity (ESC) of Various Spring Configurations.*

Type of Spring	Energy (1) Storage Capacity	Space (2) Efficiency	Notes	Typical Amounts of Energy Stored in Spring Space Envelope	
				J/mm <sup>3</sup>	ft-lbf/in <sup>3</sup>
Compression or Extension (round wire)	$\frac{S^2}{4G}$	$\frac{\pi C}{(C + 1)^2}$	(3)	$1.5-15 \times 10^{-4}$	1.8-18
Compression or Extension (square wire)	$\frac{S^2}{6.5G}$	$\frac{4C}{(C + 1)^2}$	(3)	$1.0-10 \times 10^{-4}$	1.2-12
Rectangular Cantilever & Simply Supported Beam	$\frac{S^2}{18E}$	—	—	—	—
Cantilever Beam— Triangular Plan	$\frac{S^2}{6E}$	—	—	—	—
Helical Torsion Spring (round wire)	$\frac{S^2}{8E}$	$\frac{\pi C}{(C + 1)^2}$	—	$1.0-5 \times 10^{-4}$	1.2-6
Helical Torsion Spring (square wire)	$\frac{S^2}{6E}$	$\frac{4C}{(C + 1)^2}$	—	$1.5-8 \times 10^{-4}$	1.8-9.7
Spiral Torsion (round wire)	$\frac{S^2}{8E}$	—	—	—	—
Belleville Washer	$\frac{S^2}{10E}$ to $\frac{S^2}{40E}$	0.6-0.9	(4)	$0.5-5 \times 10^{-4}$	0.6-6
Power Spring	—	0.4-0.6	(5)	$10-17 \times 10^{-4}$	12-20
Prestressed Power Spring	—	0.4-0.6	(5)	$25-30 \times 10^{-4}$	30-35
Typical Lead Acid Battery	—	—	—	$2,500-3,300 \times 10^{-4}$	3,000-4,000

(1) Energy storage capacity =  $\frac{1}{V} \int_0^f kf df$ , where V = volume of active spring material. Note that stress correction factors due to spring geometry have been omitted.

(2) Space efficiency is defined as volume of active spring material V divided by the space envelope of the spring at maximum deflection.

(3) Space efficiency does not apply to extension springs.

(4) Ratio of O.D. to I.D. of 2 is preferred for most designs.

(5) For most efficient design, the amount of space occupied by spring material equals half of the space occupied by the spring in the free position. Because of friction, difficulty in estimating the amount of active material and number of turns in the free position, determine the ESC by estimating or measuring the area under the torque revolution curve.



# 1 SPRING MATERIALS

## 1.1 Chemical and Physical Characteristics

While certain materials have come to be regarded as spring materials, they are not specially designed alloys. Spring materials are high strength alloys that often exhibit the greatest strength in the alloy system. In steels, for example, medium- and high-carbon steels are regarded as spring materials. Beryllium copper is frequently specified when a copper base alloy is required. For titanium, cold-worked and aged Ti-13V-11Cr-3Al is used. The ESC of a spring

is proportional to the square of the maximum operating stress level divided by the modulus. An ideal spring material has high strength, a high elastic limit, and a low modulus. Because springs are resilient structures designed to undergo large deflections, spring materials must have an extensive elastic range. Other factors such as fatigue strength, cost, availability, formability, corrosion resistance, magnetic permeability, and electrical conductivity can also be important and must be considered in light of cost/benefit. Consequently, careful selections must be made to obtain the best compromise.

*Table 3-1. Typical Properties of Common Spring Materials.*

Common Name	Young's Modulus E (1)		Modulus of Rigidity G (1)		Density (1) g/cm <sup>3</sup> (lb/in <sup>3</sup> )	Electrical Conductivity (1) % IACS	Sizes Normally Available (2)		Typical Surface Quality (3)	Maximum Service Temperature (4)	
	MPa 10 <sup>3</sup>	(psi) 10 <sup>6</sup>	MPa 10 <sup>3</sup>	(psi) 10 <sup>6</sup>			Min. mm (in.)	Max. mm (in.)		°C	°F
Carbon Steel Wires:											
Music (5)	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.10 (0.004)	6.35 (0.250)	a	120	250
Hard Drawn (5)	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.13 (0.005)	16 (0.625)	c	150	250
Oil Tempered	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.50 (0.020)	16 (0.625)	c	150	300
Valve Spring	207	(30)	79.3	(11.5)	7.86 (0.284)	7	1.3 (0.050)	6.35 (0.250)	a	150	300
Alloy Steel Wires:											
Chrome Vanadium	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.50 (0.020)	11 (0.435)	a,b	220	425
Chrome Silicon	207	(30)	79.3	(11.5)	7.86 (0.284)	5	0.50 (0.020)	9.5 (0.375)	a,b	245	475
Stainless Steel Wires:											
Austenitic Type 302	193	(28)	69.0	(10.)	7.92 (0.286)	2	0.13 (0.005)	9.5 (0.375)	b	260	500
Precipitation Hardening 17-7 PH	203	(29.5)	75.8	(11)	7.81 (0.282)	2	0.08 (0.002)	12.5 (0.500)	b	315	600
NiCr A286	200	(29)	71.7	(10.4)	8.03 (0.290)	2	0.40 (0.016)	5 (0.200)	b	510	950
Copper Base Alloy Wires:											
Phosphor Bronze (A)	103	(15)	43.4	(6.3)	8.86 (0.320)	15	0.10 (0.004)	12.5 (0.500)	b	95	200
Silicon Bronze (A)	103	(15)	38.6	(5.6)	8.53 (0.308)	7	0.10 (0.004)	12.5 (0.500)	b	95	200
Silicon Bronze (B)	117	(17)	44.1	(6.4)	8.75 (0.316)	12	0.10 (0.004)	12.5 (0.500)	b	95	200
Beryllium Copper	128	(18.5)	48.3	(7.0)	8.26 (0.298)	21	0.08 (0.003)	12.5 (0.500)	b	205	400
Spring Brass, CA260	110	(16)	42.0	(6.0)	8.53 (0.308)	17	0.10 (0.004)	12.5 (0.500)	b	95	200
Nickel Base Alloys:											
Inconel® Alloy 600	214	(31)	75.8	(11)	8.43 (0.304)	1.5	0.10 (0.004)	12.5 (0.500)	b	320	700
Inconel Alloy X750	214	(31)	79.3	(11.5)	8.25 (0.298)	1	0.10 (0.004)	12.5 (0.500)	b	595	1100
Ni-Span-C®	186	(27)	62.9	(9.7)	8.14 (0.294)	1.6	0.10 (0.004)	12.5 (0.500)	b	95	200
Monel® Alloy 400	179	(26)	66.2	(9.6)	8.83 (0.319)	3.5	0.05 (0.002)	9.5 (0.375)	b	230	450
Monel Alloy K500	179	(26)	66.2	(9.6)	8.46 (0.306)	3	0.05 (0.002)	9.5 (0.375)	b	260	500

Carbon Steel Strip:												
AISI 1050	207	(30)	79.3	(11.5)	7.86	(0.284)	7	0.25 (0.010)	3 (0.125)	b	95	200
1065	207	(30)	79.3	(11.5)	7.86	(0.284)	7	0.08 (0.003)	3 (0.125)	b	95	200
1074, 1075	207	(30)	79.3	(11.5)	7.86	(0.284)	7	0.08 (0.003)	3 (0.125)	b	120	250
1095	207	(30)	79.3	(11.5)	7.86	(0.284)	7	0.08 (0.003)	3 (0.125)	b	120	250
Bartex®	207	(30)	79.3	(11.5)	7.86	(0.284)	7	0.10 (0.004)	1 (0.040)	a	95	200
Stainless Steel Strip:												
Austenitic Types	193	(28)	69.0	(10)	7.92	(0.286)	2	0.08 (0.003)	1.5 (0.063)	b	315	600
301, 302												
Precipitation Hardening 17-7 PH	203	(29.5)	75.8	(11)	7.81	(0.282)	2	0.08 (0.003)	3 (0.125)	b	370	700
Copper Base Alloy Strip:												
Phosphor Bronze (A)	103	(15)	43	(6.3)	8.86	(0.320)	15	0.08 (0.003)	5 (0.188)	b	95	200
Beryllium Copper	128	(18.5)	48	(7.0)	8.26	(0.298)	21	0.08 (0.003)	9.5 (0.375)	b	205	400

(1) Elastic moduli, density and electrical conductivity can vary with cold work, heat treatment and operating stress. These variations are usually minor but should be considered if one or more of these properties is critical.

(2) Diameters for wire; thicknesses for strip.

(3) Typical surface quality ratings. (For most materials, special processes can be specified to upgrade typical values.)

a. Maximum defect depth: 0 to 0.5% of d or t.

b. Maximum defect depth: 1.0% of d or t.

c. Defect depth: less than 3.5% of d or t.

(4) Maximum service temperatures are guidelines and may vary due to operating stress and allowable relaxation.

(5) Music and hard drawn are commercial terms for patented and cold-drawn carbon steel spring wire.

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Table 3-1 lists some commonly used alloys along with data for material selection purposes. Data on mechanical properties are presented in the "Spring Wire" and "Spring Strip" subsections. Specifications have been written by many national and international organizations. These specifications are cross-referenced to Associated Spring specifications in Table 3-2. However, correlation between the specifications is only approximate. Associated Spring specifications were developed exclusively for high quality material for spring applications, and they are generally more detailed and stringent than other specifications.

Surface quality has a major influence on fatigue strength and is often not clearly delineated on national specifications. It is important to use only those materials with the best surface integrity for fatigue applications, particularly those in the high cycle region.

In steel alloys, for which processing costs are a large fraction of product cost, surface quality can vary over an appreciable range. Depth of surface imperfections, such as seams, pits, and die marks, can be up to 3.5% of diameter for commercial spring wire grades (ASTM A-227 and A-229). Various intermediate qualities can



be obtained. Highest levels are represented by music and valve spring quality grades, which are virtually free of surface imperfections. Decarburization, which can also adversely affect fatigue performance, follows a similar pattern. Surface quality of spring materials is a function of the care exercised in their production and the processes employed. Materials produced with a high level of surface integrity are more costly than commercial grades.

## 1.2 Elastic Modulus

The modulus of elasticity in tension and shear is vital to spring design. Table 3-1 lists recommended values for commonly used spring alloys. For most steels and age-hardenable alloys, the modulus varies as a function of chemical composition, cold work, and degree of aging. Usually, variations are small and can be compensated for by adjustment of reference parameters of the spring design (e.g., number of active coils and coil diameter).

For most materials, moduli are temperature-dependent and vary inversely with temperature by approximately 2% per 55°C (100°F). Since nonambient temperature testing is costly, design criteria should be specified at room temperature after having made appropriate compensation for the application temperature. Certain nickel-chromium-iron alloys are designed to have a constant modulus over the temperature range from -5° to 65°C (-50° to 150°F) and are exceptions to the above rule.

For true isotropic materials, the elastic moduli in tension (E) and shear (G) are related through Poisson's ratio by the following expression, so that for common spring materials, any one of the parameters may be approximated using the other two:

$$\mu = \frac{E}{2G} - 1$$

## 1.3 Magnetic Characteristics

For most applications, the question of "magnetic or not" is adequately answered with the use of a permanent magnet. For some applications, even very low levels of magnetic behavior can be detrimental. Then, it is desirable to know the magnetic permeability of candidate materials and reach agreement between parties on a maximum allowable value. Table 3-3 lists approximate values for a number of low permeability materials along with other frequently used alloys.

Since permeability can be altered by cold work, some variation can be expected. In general, low permeability materials are more expensive, so designers should specify low levels only when absolutely necessary. Often, nitrogen-strengthened manganese stainless steels are good choices because they have good strength at moderate cost.

## 1.4 Heat Treatment of Springs

Heat-treating temperatures for springs can be divided into two ranges. Low temperature heat treatments in the 175° to 510°C (347° to 950°F) range are applied to springs after forming to reduce residual stresses and stabilize parts dimensionally. For carbon steels, stainless steels, and some age-hardenable alloys, low temperature heat treatments are used to increase or restore the set point. Electroplated carbon steel parts are heat-treated at low temperatures prior to plating, and baked afterward to reduce the susceptibility to hydrogen embrittlement. Most low temperature stress relieving and age-hardening of springs are done in air, and a moderate amount of oxide may be formed on the part. No detrimental effects of this oxide have been noted.

High temperature heat treatments are used to strengthen annealed material after spring forming. High- carbon steels are strengthened by austenitizing in the temperature range 760° to 900°C (1480° to 1652°F), quenching to form martensite and then tempering. Some nickel base alloys are strengthened by high temperature aging treatments. Because substantial oxidation occurs at these elevated temperatures, it is advisable to prevent excessive oxidation by using an appropriate protective atmosphere.

Heat treatments suitable for many commonly used materials are listed in Table 3-4. Selection of a temperature within a given range can only be made after considering the material,

size, strength level, application conditions, and desired characteristics. For additional guidance, Associated Spring engineers should be consulted. Unless otherwise noted, 20 to 30 minutes of exposure at temperature is sufficient to obtain the bulk of the stress-relieving effect.

Many spring-like parts involve forms that preclude the use of pre-hardened material. In these cases, soft or annealed material must be used and heat-treated to attain spring properties after forming. Thin high- carbon and alloy steel parts become distorted when hardened by quenching. Distortion may be reduced by fixture tempering; however, this process is costly and should be avoided if possible by using pre-tempered materials.

Table 3-2. Related Spring Material Specifications.

Associated Spring Specs	Common Trade Names	SAE	ASTM	AMS	Military	Federal	British EN	BS	German DIN	Japanese JIS
<b>Spring Wire</b>										
AS-5	Music Wire	1085 J178	A228	5112	S-46049	QQW-470 (obsolete)		1408 or 5216 5201	17223, Sheet 1, 1.1200	G3522, SWP-A, B, V
AS-10	Oil Tempered Carbon Steel	1066 J316	A229			QQW-428		2803, grade 3	17223, Sheet 2, 1.1230	G3560, SWO-A, B
AS-20	Cold Drawn Carbon Steel	1066 J113	A227				49B	1408 5216 NS or HS	17222, Sheet 1, 1.0500	G3521, SW-A, B, C
AS-25	Oil Tempered Carbon Steel*	1070	A230	5115				2803, Grades 1&2	17223, Sheet 2	G3561, SWO-V
AS-32	Oil Tempered Chrome Vanadium*	6150 J132	A232	6450	W-22826	QQW-412	47 50		17225, 50CrV4	G3565, SWOCV-V
AS-33	Oil Tempered Chrome Silicon*	9254 J157	A401			QQW-412	48A		17225, 67SiCr5	G3566, SWOSC-V
AS-35	Stainless Steel	30301 30302 J230	A313; Type 301, Type 302	5688		QQW-423 (obsolete)	58A	2056	1.4300 1.4310 17224	G4314, SUS 302
AS-36	17-7 PH	J217	A313; Type 631	5678					17224, 1.4568	G4314, SUS 631J1
AS-44	Inconel X-750			5698, 5699						
AS-45	Copper Beryllium	CA-172	B197	4725, Cond. A		QQW-530, Cond. A		2873, CB101	17666, 2.1247.55	
AS-55	Spring Brass	CA-260	B134, #260			QQW-321, #260		2786, CZ 107	17660, 2.0265	
AS-60A	Phosphor Bronze	CA-510	B159, #510	4720				2873, PB102	17662, 2.1030.39	
AS-60C	Phosphor Bronze	CA-521	#521							



AS-70	Chromium Steel	5160H	A304 A689					970, Part 5			
<b>Spring Strip</b>											
AS-100		1095	A682 A684	5121 5122	S-7947 Annealed Cold-Rolled			44D	1449, Part 3B, CS100	17232, 1.1274	G3311, SK4M
AS-101		1074 1075	A682 A684	5120				42E	1449, Part 3B, CS, CS80	17222, 1.1210	G3311, 575CM
AS-102		1050	A682 A684	5085					1449, Part 3B, CS50		G3311, S50CM
AS-103		1065	A682 A684	5115				42F	1440, Part 3B, CS60, CS70	17222, 1.1230	G3311, S65CM
AS-105	Bartex	1085									
AS-135-A AS-135-B	Stainless Steel	30301 30302	A177	5517 5518, 5519	S-5059	QQS-766	58A	1449, Part 4, 302-S-25	17224, 1.4310 1.4300		G4313, SUS-301-CSP SUS-302-CSP
AS-136	17-7 PH			5528 5529 Spring Temper	S-25043 Cond. A					17224, 1.4568	G4313, SUS-631-CSP
AS-144	Inconel X-750			5542	N 7786						
AS-145	Copper	CA-172	B194, 172	4530(AT) 4532 (1/2HT)		QOC-533		2870, CB101		17666, 2.1247.55	
AS-155	Spring Brass	CA-260	B36,#26	4507, 1/2H		QQB-613, Comp. 2 1/2H		2870, CZ108		17660, 2.0265	
AS-160A	Phosphor Bronze	CA-510	B103, #510	4510 Spring Temper		QQB-750, Comp. A		2870, PB102		17662, 2.1030.39	
AS-160C	Phosphor Bronze	CA-521	B103, 3#52								

**Table 3-3. Magnetic Characteristics of Some Materials.**

Materials	Permeability at 200 Oersted, Room Temperature
Air	1
Brasses, Bronzes	Nonmagnetic
Carbon Steels	> 500
Elgiloy®	1.000035
Inconel Alloys:	
600	1.01
625	1.0006
X-750	1.0035
Stainless Steels:	
Type 301, spring temper	> 30
Type 302, spring temper	> 12
631 (17-7 PH)	> 40
XM-28: Nitronic® 32*	1.011
Nitronic 50*	1.004
Titanium Alloys	Nonmagnetic

\*Nitrogen-strengthened manganese stainless steels.

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**Table 3-4. Typical Heat Treatments for Springs After Forming.**

Materials	Heat Treatment	
	°C	°F
Patented and Cold-Drawn Steel Wire	190-230	375-450
Tempered Steel Wire:		
Carbon	260-400	500-750
Alloy	315-425	600-800
Austenitic Stainless Steel Wire	230-510	450-950
Precipitation Hardening Stainless Wire (17-7 PH):		
Condition C	480/1 hour	900/1 hour
Condition A to TH 1050	760/1 hour cool to 15°C followed by 565/1 hour	1400/1 hour, cool to 60°F followed by 1050/1 hour
Monel:		
Alloy 400	300-315	575-600
Alloy K500, Spring Temper	525/4 hours	980/4 hours
Inconel:		
Alloy 600	400-510	750-950
Alloy X-750:		
#1 Temper	730/16 hours	1350/16 hours
Spring Temper	650/4 hours	1200/4 hours
Copper Base, Cold Worked (Brass, Phosphor Bronze, etc.)	175-205	350-400
Beryllium Copper:		
Pretempered (Mill Hardened)	205	400
Solution Annealed, Temper Rolled or Drawn	315/2-3 hours	600/2-3 hours
Annealed Steels:		
Carbon (AISI 1050 to 1095)	800-830*	1475-1525*
Alloy (AISI 5160H 6150, 9254)	830-885*	1525-1625*

\*Time depends on heating equipment and section size. Parts are austenitized then quenched and tempered to the desired hardness.

Tempering is an effective stress-relieving treatment and results in negligible levels of residual stress. Some spring materials, such as beryllium copper and 17-7 PH, are strengthened after forming by age hardening. This provides a good stress relief, but may also cause distortion unless special techniques are used.

### 1.5 Environmental Considerations

Frequently, operating environment is the single most important consideration for proper spring material selection. For successful application, material must be compatible with the environment and withstand effects of temperature and corrosion without an excessive loss in spring performance. Corrosion and elevated temperatures decrease spring reliability. The effect of temperature on spring materials is predictable and discussed below. Compatibility of spring materials and spring coating systems with corrosive environments is discussed in general terms. For specific applications, the designer is urged to rely upon previous experience or consult with Associated Spring engineers.

### 1.6 Stress Relaxation

Primary concern for elevated temperature applications of springs is stress relaxation. Stress relaxation is the loss of load or available deflection that occurs when a spring is held or cycled under load. Temperature also affects modulus as well as tensile and fatigue strength. For a given spring, variables that affect stress relaxation are stress, time, and temperature, with increases in any parameter tending to increase the amount of relaxation. Stress and temperature are related exponentially to relaxation. Curves of relaxation versus these



parameters are concave upward as is shown in Figures 3-1 and 3-2. Other controllable factors affecting relaxation include the following:

- Alloy type: More highly alloyed materials are generally more resistant at a given temperature or can be used at higher temperatures.
- Residual stress: Residual stresses remaining from forming operations are detrimental to relaxation resistance. Therefore, use of the highest practical stress-relief temperatures is beneficial. Shot-peening is also detrimental to stress relaxation resistance.
- Heat setting: Various procedures can be employed to expose springs to stress and heat for varying times to prepare for subsequent exposures. Depending on the method used, the effect is to remove a usually large first-stage relaxation and/or to establish a residual stress system that will lessen relaxation influences. In some cases, the latter approach can be so effective that in application, compression springs may “grow” or exhibit negative relaxation. Increase in free length does not usually exceed 1% to 2%.
- Grain size: Coarse grain size promotes relaxation resistance. This phenomenon is used only in very high temperature applications.

Because so many variables are involved, it is impossible to cite comprehensive data in a publication of this type, but Table 3-1 does show approximate maximum service temperatures for many commonly used materials. It should be remembered that if a material is used at its maximum temperature, a substantial reduction must be made in applied stress from that used at room temperature.

Fig. 3-1. Relaxation versus Initial Stress for Spring Materials.

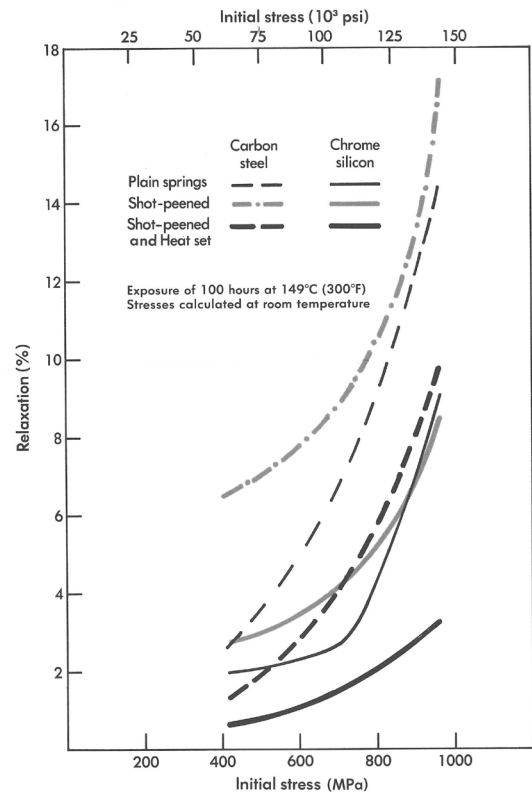
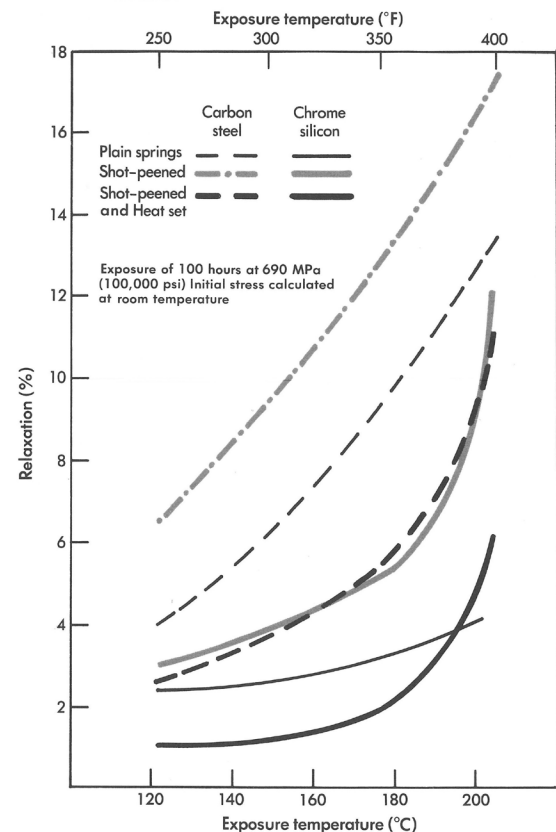


Fig. 3-2. Relaxation versus Temperature for Spring Materials.



## 1.7 Corrosion

The effect of a corrosive environment on spring performance is difficult to predict with certainty. General corrosion, galvanic corrosion, stress corrosion, and corrosion fatigue reduce life and load-carrying ability of springs. The two most common methods employed to combat effects of corrosion are to specify materials that are inert to the environment and to use protective coatings. Use of inert materials affords the most reliable protection against deleterious effects of all types of corrosion; however, this is often costly and sometimes impractical. Protective coatings are often the most cost-effective method to prolong spring life in corrosive environments. In special situations, shot-peening can be used to prevent stress corrosion, and cathodic protection systems can be used to prevent general corrosion.

**Table 3-5. Guide for Selecting Minimum Thicknesses for Zinc and Cadmium Coatings.**

Zinc on Iron and Steel Part per QQ-Z-325 (1)				Cadmium on Iron and Steel Parts per QQ-P-416 (2)		
Minimum Recommended Thickness mm (in.)	Finish Type	Salt Spray Test, Hours to Corrosion		Minimum Thickness mm (in.)	Finish Type	Salt Spray Test, Hours to White Corrosion
		White	Red			
0.025 (0.0010)	A	—	192	0.013 (0.00050)	B	96
	B	96	—			
	C	—	192			
0.013 (0.00050)	A	—	—	0.008 (0.00030)	B	96
	B	96	—			
	C	—	—			
0.005 (0.00020)	A	—	36	0.005 (0.00020)	B	96
	B	96	—			
	C	—	36			

(1) Requirements for zinc coating (electrodeposited).

(2) Requirements for cadmium plating (electrodeposited).

Finish Type:

- A. Without supplementary chromate or phosphate treatment.
- B. With supplementary chromate treatment.
- C. With supplementary phosphate treatment.

Coatings may be classified as galvanically sacrificial or simple barrier coatings. Sacrificial coatings for high carbon steel substrates include zinc, cadmium (and alloys thereof), and, to a lesser degree, aluminum. Due to its toxicity,

cadmium coating should be specified only when absolutely necessary. Because sacrificial coatings are chemically less noble than steel, the substrate is protected in two ways. First, the coating acts as a barrier between substrate and environment. Second, galvanic action between coating and substrate cathodically protects the substrate. This characteristic allows sacrificial coatings to continue their protective role even after the coating is scratched, nicked, or cracked. The amount of damage a sacrificial coating can sustain and still protect the substrate is a function of the size of the damaged area and the efficiency of the electrolyte involved. The salt spray life criteria for three thicknesses of sacrificial coatings are shown in Table 3-5. Use of conversion coatings, such as chromates, lengthens the time of protection by protecting sacrificial coatings. Salt spray (fog) is an accelerated test, and results may or may not correlate with corrosive activity in the actual environment. The test is useful as a control to ensure the coating was applied properly.

Metallic coatings are normally applied by electroplating. Since most high hardness steels are inherently very susceptible to hydrogen embrittlement, plating must be carried out with great care to minimize embrittlement and subsequent delayed fracture. A baking operation after plating is also essential. The designer should observe these points during design and specifications:

- Minimize sharp corners and similar stress-concentration points in design.
- Keep hardness as low as possible.
- Keep operating stress down, in accordance with lowered hardness value.
- Specify plating thickness, depending upon requirements.

- Specify that parts be baked after plating.
- Consider use of HEP® strips to monitor the plating operation.

Residual stress from forming operations must be reduced by stress relief at the highest practical temperature. Otherwise, the combined effect of residual tension and hydrogen absorbed during plating can induce cracking even before plating is complete.

Similar cautions apply if acid cleaning procedures are contemplated.

Mechanical plating provides an effective means of zinc or cadmium protection with minimum hydrogen embrittlement. It is particularly recommended where parts have high residual stress, have been hardened above HRC48, and are used with high static loads. The process can be applied only to parts that do not tangle and have a clean, fully accessible surface. Hydrogen embrittlement, although unlikely, is still possible if parts are cleaned by pickling. When appropriate, coatings of zinc, tin, cadmium, or an alloy of cadmium can be applied by mechanical plating processes.

Cadmium, zinc, or, more commonly, alloys of the two can be applied to steel spring wire during its production, and under some circumstances this alternative is highly desirable. It is best suited to small diameter wire and, in general, for the production of springs not requiring grinding.

Springs are almost always in contact with other metal parts. In a corrosive environment, it is important that the spring material be more noble than components in contact with it. Table 3-6 shows a partial list of alloys in increasing order of nobility. When any two alloys are placed in contact in the presence of an electrolyte, the less noble alloy (higher on the list) will be attacked. The attack will be significantly more vigorous than that of the electrolyte acting by itself.

**Table 3-6. Order of Nobility.**

<b>Galvanic Series with electrolyte such as seawater.</b>	
Magnesium	Least noble (+), Anodic
Zinc	
Aluminum	
Cadmium	
Steel or Iron	
Cast Iron	
Stainless Steel, series 300 (active)	
Hastelloy® C	
Nickel (active)	
Inconel (active)	
Hastelloy B	
Brasses, Bronzes	
Monel	
Nickel (passive)	
Inconel (passive)	
Stainless Steel, series 300 (passive)	
Titanium	Most noble (-), Cathodic

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The list of coatings, which protect the base material by acting as a barrier to the environment, is extensive and increases as new finishes and techniques are developed. Table 3-7 shows protection available from some of the common barrier finishes. This information is not for selection purposes; it simply shows the range of protection afforded. In fact, the hours of salt spray protection may only be valid for the specimen and test conditions employed in this series of tests. The tests were conducted on springs, which had undergone 4 million cycles in a fatigue test prior to salt spray exposure.

While coatings frequently increase in effectiveness as their thicknesses are increased, cautions are in order. Tendencies to crack increase as coating thickness increases, and the coating increases the size of the spring. For example, coatings increase the solid height and diametral clearances required for compression springs. Brittle coatings such as epoxy can chip under impact, leaving unprotected spots. Tough coatings such as vinyl resist chipping, but

bruises and tears of abrasions can expose the base material and trap corrosive agents. This allows corrosion to continue after exposure, and in these circumstances, coated springs occasionally exhibit shorter lives than uncoated springs.

Frequently oils, waxes, or greases provide adequate protection. Effectiveness of these coatings is often dependent on the nature of the surface to be protected. In general, lustrous or smooth parts will not retain oils, and waxes, paraffin-based oils, or greases are recommended. Steels can be phosphate coated by a conversion process. Phosphate coatings have a high retention for oils, greases, or paints. The combination of a phosphate and oil coating becomes a corrosion inhibitor more effective than either of the components. A similar effect is obtained by retaining or deliberately forming oxides on metal surfaces to hold corrosion inhibitors or lubricants. Oil-tempered spring wire is a notable example of this technique.

## 1.8 Spring Wire

Tensile properties of spring wire vary with size (Figure 3-3). Common spring wires with the highest strength are ASTM 228 and ASTM 401 materials. ASTM A313 Type 302, A232, and A230 materials have slightly lower tensile strength with surface qualities suitable for fatigue applications. Hard-drawn (ASTM 227) and oil-tempered (ASTM 229) are also supplied at lower strength levels and are most suitable for static applications.

Most spring wires can be wrapped on their own diameter (bent around a pin with a diameter equal to the wire diameter). Exceptions include some copper-base alloys and large diameter and/or high strength alloys. Because stress relieving increases yield strength of cold-drawn

spring wire, all sharp bends of this grade material should be made prior to stress relief.

*Table 3-7. Salt Spray Resistance of Common Barrier Finishes.*

Protective Material	Standard Salt Spray Test Resistance, hours	Description
Paints:		
Japan	15–20	Dark colored, usually dipped, cured by baking.
Lacquer	30–100	Usually applied by spraying. Air dried.
Enamel Paint	50–400 25–300	Hard finish; applied by spray, brush or dip; cured by air or baking.
Oils, waxes	1–300	Lubricating, rust-inhibiting, hard drying and nondrying oils.
Phosphates with supplemental oils, waxes, etc.	24–600	Chemical treatment converting steel surface to iron phosphate crystalline surface. Affords a bond for oils and paints.
Cadmium, zinc	24–100	Electroplated or mechanically plated.

This information is based on laboratory-controlled applications and test. The protective material selected, cleanliness of parts, method of application, subsequent operations and part usage affect performance. The choice of a spring finish must also consider shipping, assembly, end use and total cost.

## 1.9 Cost and Availability

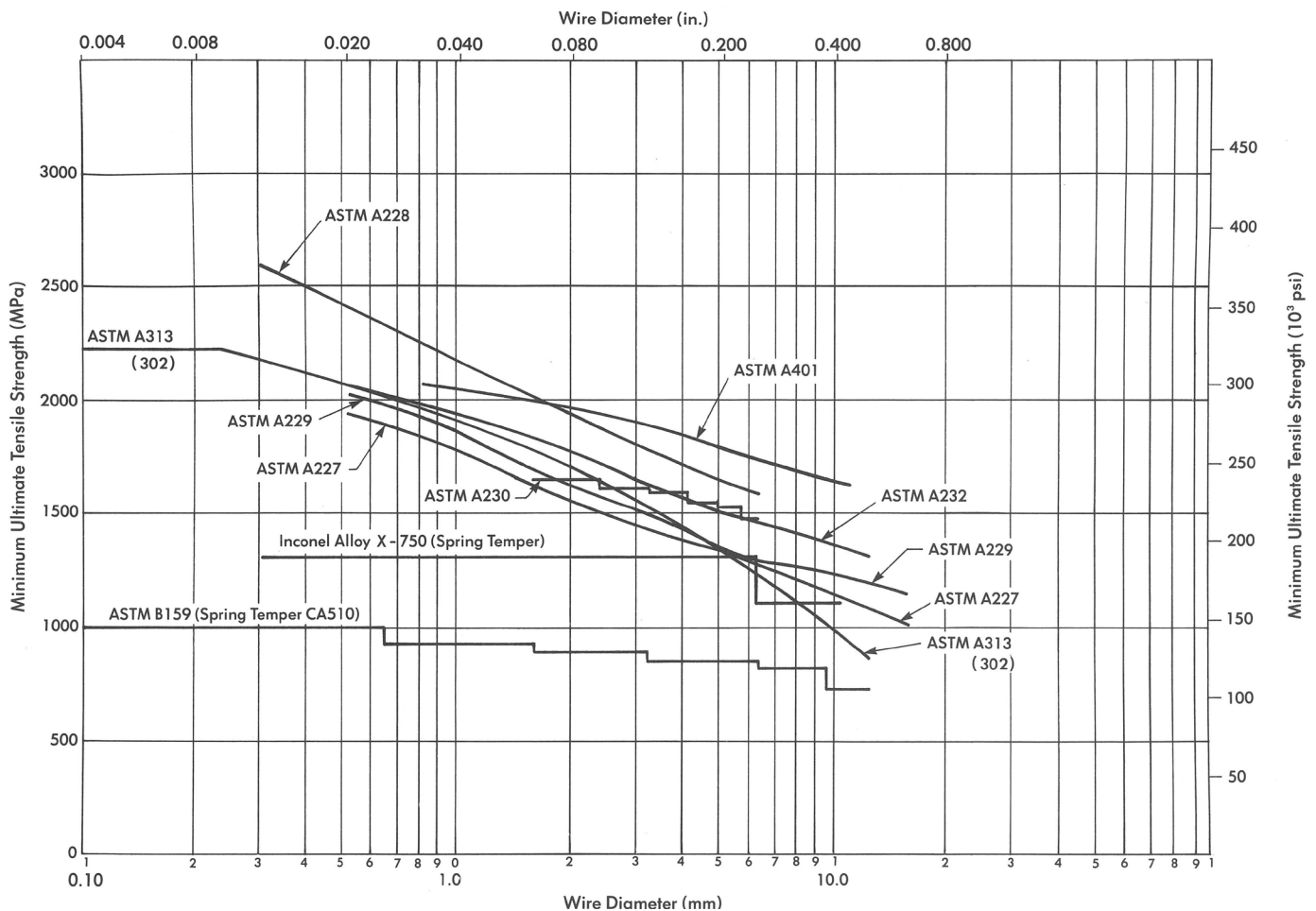
Cost and availability of spring wire have an important bearing on custom-made springs. Frequently, the designer can minimize cost and improve delivery by selecting preferred sizes of wire (Table 3-8). Associated Spring stocks a large inventory of preferred sizes and should be consulted especially when quantities are relatively low. Since absolute costs change very quickly, material costs are presented on a relative basis. Changes in ranking with the passage of time are possible. Other factors such as wire size, surface quality, and quantity also affect price per pound. Table 3-9 shows data for quantities large enough to avoid extra costs for 2 mm (0.079") wire. For all materials, processing to small sizes adds significantly to basic alloy cost. Production of high-quality

surfaces may carry a cost premium. As the base alloy value increases, these factors, though still important, represent a smaller fraction of the total raw material cost. Patented and cold-drawn wire (ASTM A227) is used as a base with an assigned value of 1.0. When the amount of material required is large, choice of material is very important, for it will usually be the major element in the spring cost. If the amount of material is small and particularly if quick delivery is important, immediate availability rather than cost often dictates the material choice.

tolerances on spring wire are critical. Standard tolerances are shown in Table 3-10. Closer tolerances are available on request.

Springmaking operations rarely change the diameter of the material. Since spring properties are often dependent upon diameter to the fourth power or thickness to the third power,

Fig. 3-3. Minimum Tensile Strengths of Spring Wire.



**Table 3-8. Preferred Diameters for Spring Steel Wire.**

First Preference	Metric Sizes (mm)		English Sizes (in.)	
	Second Preference	Third Preference	First Preference	Second Preference
0.10			0.004	
	0.11		0.005	
0.12			0.006	
	0.14		0.008	
0.16				0.009
	0.18		0.010	
0.20				0.011
	0.22		0.012	
0.25				0.013
	0.28		0.014	
0.30				0.015
	0.35		0.016	
0.40				0.017
	0.45		0.018	
0.50				0.019
	0.55		0.020	
0.60				0.021
	0.65		0.022	
	0.70		0.024	
0.80			0.026	
	0.90		0.028	
1.0			0.030	
	1.1			0.031
1.2				0.033
	1.4	1.3	0.035	
			0.038	
1.6				0.040
	1.8		0.042	
2.0			0.045	
	2.2			0.047
		2.4	0.048	
2.5			0.051	
			0.055	
		2.6	0.059	
	2.8		0.063	
3.0			0.067	
		3.2	0.072	
	3.5		0.076	
		3.8	0.081	
4.0			0.085	
		4.2	0.092	
	4.5		0.098	
		4.8		0.102
5.0			0.105	
	5.5		0.112	
6.0				0.120
	6.5		0.125	
	7.0			0.130
		7.5	0.135	
8.0				0.140
		8.5	0.148	
	9.0			0.156
		9.5	0.162	
10.0				0.170
	11.0		0.177	
12.0			0.192	
	13.0			0.200
14.0			0.207	
	15.0			0.218
			0.225	
16.0			0.250	
				0.262
			0.281	
				0.306
			0.312	
			0.343	
			0.362	
			0.375	
			0.406	
			0.437	
			0.469	
			0.500	

### 1.10 Spring Strip

Most flat springs are made from AISI grades 1050, 1065, 1074, and 1095 steel strip. These compositions are listed in ASTM specifications A682 and A684.

Tensile strength and formability characteristics are shown in Figure 3-4. The vertical inclined bands delineate three strength levels as functions of stock thickness and hardness. Horizontal curves indicate minimum bending radii required for the strength levels they intersect. Interpolations can be made between any two bands or lines for intermediate levels. Formability criteria are given for relatively smooth bends made at reasonable bending rates. Operations that apply forming forces other than smooth bending, or have impact characteristics, may require larger radii to prevent fracture. Four-slid part manufacture, progressive die work, and secondary forming are examples of operations that often produce less-than-ideal bending.

**Table 3-9. Ranking of Relative Costs of Common Spring Wires.**

Wire	Specification	Relative Cost of 2 mm (0.079") Dia.	
		Mill Quantities	Ware-House Lots
Patented and Cold Drawn Oil Tempered	ASTM A227	1.0	1.0
	ASTM A229	1.3	1.3
Music Carbon Valve Spring	ASTM A228	2.6	1.4
	ASTM A230	3.1	1.9
Chrome Silicon Valve Stainless Steel (Type 302)	ASTM A401	4.0	3.9
	ASTM A313 (302)	7.6	4.7
Phosphor Bronze Stainless Steel (Type 631) (17-7 PH)	ASTM	8.0	6.7
	ASTM A 313 (631)	11	8.7
Beryllium Copper Inconel Alloy X-750	ASTM B197	27	17
		44	31



**Table 3-10. Standard Tolerances for Spring Wire.**

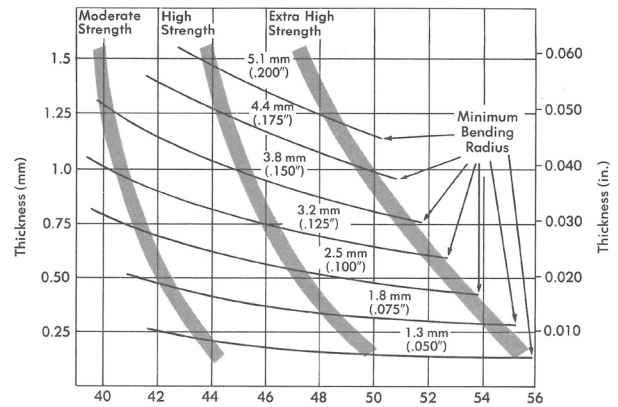
Diameter: mm (in.)	Tolerance: mm (in.)	Maximum Out-of-Roundness: mm (in.)
0.51–0.71 (0.020–0.028)	± 0.010 (0.0004)	0.010 (0.0004)
0.71–2.00 (0.028–0.078)	± 0.015 (0.0006)	0.015 (0.0006)
2.00–3.00 (0.078–0.118)	± 0.020 (0.0008)	0.020 (0.0008)
3.00–6.00 (0.118–0.240)	± 0.030 (0.00118)	0.030 (0.0012)
6.00–9.00 (0.240–0.354)	± 0.050 (0.00197)	0.050 (0.002)
9.50–16.00 (0.375–0.625)	± 0.070 (0.00276)	0.070 (0.0028)

Most spring wires can be purchased to tighter tolerances. Music wire and most nonferrous materials are regularly made to closer tolerances.

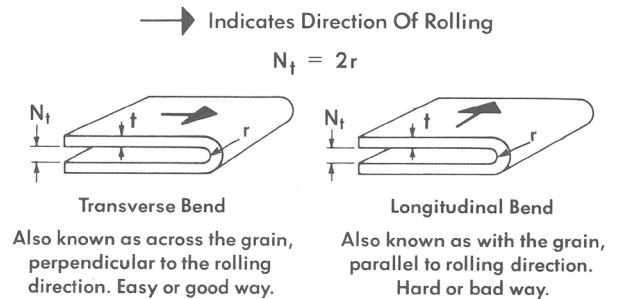
Direction of bending with respect to rolling direction is an important consideration. Formability of strip is greater in transverse than in longitudinal directions (Figure 3-5). If a part is designed with two identical bends at 90° to each other, it is common practice to orient the part so that both bends are made at 45° to rolling direction. Dimensionless parameter  $2r/t$ , often referred to as bend factor, is frequently used as a measure of formability. Materials with low values are more formable than materials with high values. This measure is only a guide, since it does not allow for tooling considerations and complex strains associated with forming operations.

Spring steels are normally produced to specified hardness levels that are related to tensile strength (Figure 3-6). Composition is not shown in Figure 3-4 because the lowest carbon level (AISI 1050) can be used at high strength levels, and the highest carbon level (AISI 1095) can be tempered to the lowest strength levels. In general, higher carbon levels are used when strength is critical and lower carbon levels when formability is critical.

**Fig. 3-4. Minimum Transverse Bending Radii for Various Tempers and Thicknesses of Tempered Spring Steel.**



**Fig. 3-5. Orientation of Bend Axis to Rolling Direction for Transverse and Longitudinal Bends.**



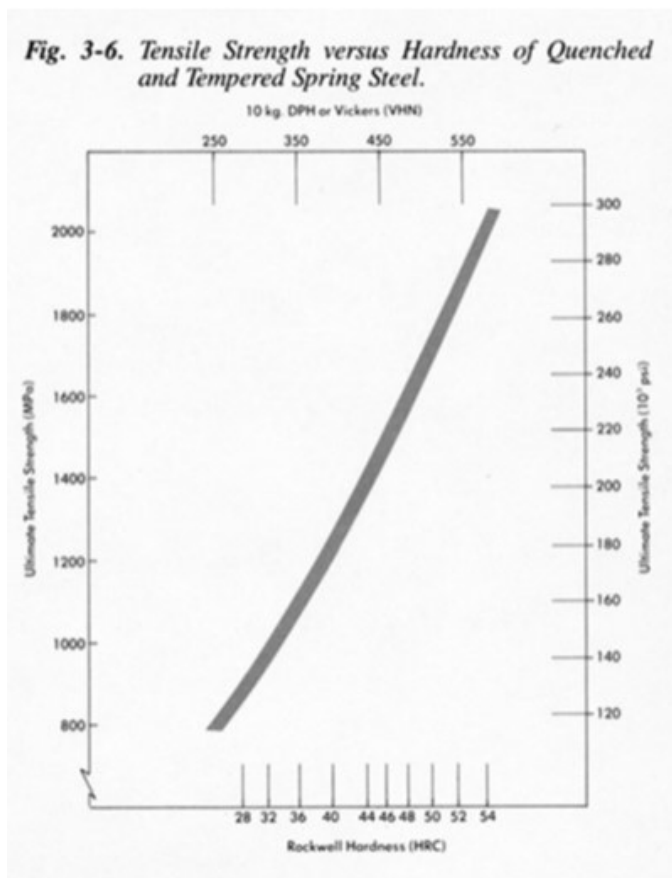
Hardness levels above HRC 50 result in high strength but are not generally recommended due to notch sensitivity. Surface and edge smoothness become critical and plated parts become highly susceptible to static fracture due to trapped hydrogen.

Parts that cannot be made within formability limits of pre-tempered strip are made from annealed strip and hardened and tempered after forming. To maintain critical dimensions, it is often necessary to fixture temper these parts. Sharp bends are not only difficult to fabricate but are also undesirable in service because of stress concentration. The formability limits of annealed spring steels are presented in Table 3-11.

In flat spring designs where the edge of the strip becomes an edge of the part, the type of edge

is important, particularly for cyclic applications. Common types of edges available are shown in Figure 3-7. Slit edge (No. 3) and deburred (No. 5) are preferred for blanked parts and static applications. No. 1 round edge is recommended for cyclic applications to reduce the stress concentration and eliminate the edge flaws due to slitting. Configurations shown in Figure 3-7 are approximate, and it is advisable to use both the numerical designation and a description when specifying edge condition.

Commercial thickness tolerances for spring steel strip are presented in Table 3-12. Many flat springs and spring washer designs can tolerate this variation. Since the load varies as the cube of the thickness, critical designs may require closer tolerances.

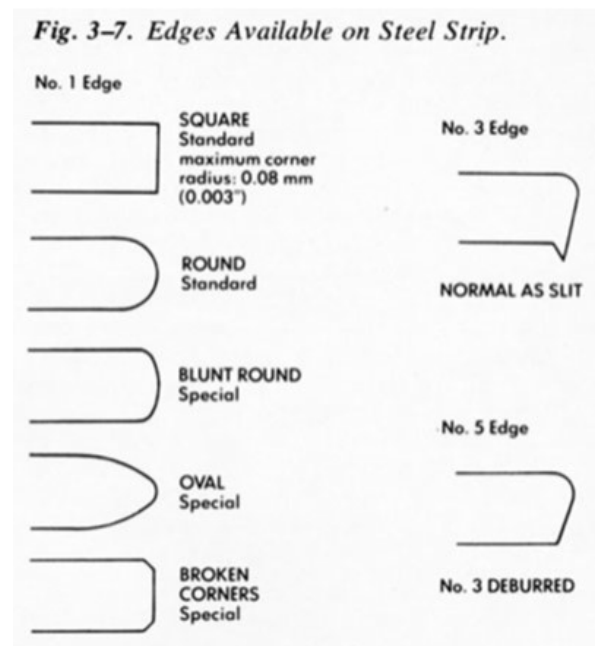


## 1.11 Other Spring Materials

A variety of materials other than carbon steel strip are used for flat springs (Table 3-13). When high conductivity is required, copper-base alloys are usually specified. Stainless steels are used in applications requiring heat or corrosion resistance. Typical tensile strength levels for these alloys after heat treatment are shown in Table 3-13. Bend factors and tensile elongations are for alloys in "as received" condition prior to final heat treatment.

## 1.12 Specifying Hardness

Hardness tests are used extensively to inspect strip and flat springs, and it is necessary to specify the correct scale. Recommended hardness scales for steels are presented in Table 3-14. To obtain accurate readings free from the effect of the anvil, it is important to limit the thickness of the material for each hardness scale as shown in Figure 3-8 for hard materials and Figure 3-9 for soft materials.



**Table 3–11. Formability of Annealed Spring Steels.**

Thickness (t) mm (in.)	Direction of Bend	AISI 1050 N <sub>t</sub> /t		AISI 1065 N <sub>t</sub> /t		AISI 1074, 1075 N <sub>t</sub> /t		AISI 1095 N <sub>t</sub> /t	
		Annealed (standard lowest max.)	Annealed (special lowest max.)*	Annealed (standard lowest max.)	Annealed (special lowest max.)*	Annealed (standard lowest max.)	Annealed (special lowest max.)*	Annealed (standard lowest max.)	Annealed (special lowest max.)*
1.9 mm (0.076)-over	⊥ 	2 4	0 3	2 4	0 3	2 4	0 3	3 5	1 4
0.9–1.89 mm (0.036–0.075")	⊥ 	1 2	0 1	1 2	0 1	1 2	0 1	2 3	0 2
0.37–0.89 mm (0.015–0.035")	⊥ 	0 1	0 0	0 1 1/2	0 1	1 1 1/2	0 1	1 2	0 1
0.2–0.36 mm (0.008–0.014")	⊥ 	0 0	0 0	0 0	0 0	1 1	0 0	1 1	0 1/2

Formability is determined by slowly bending a sample over 180° until its ends are parallel. The measured distance between the ends is N<sub>t</sub>. For example, if N<sub>t</sub> = 4 and t = 2, then N<sub>t</sub>/t = 2  
 \*Available as Barco-Form® from Wallace Barnes Steel subsidiary of Theis of America, Inc.

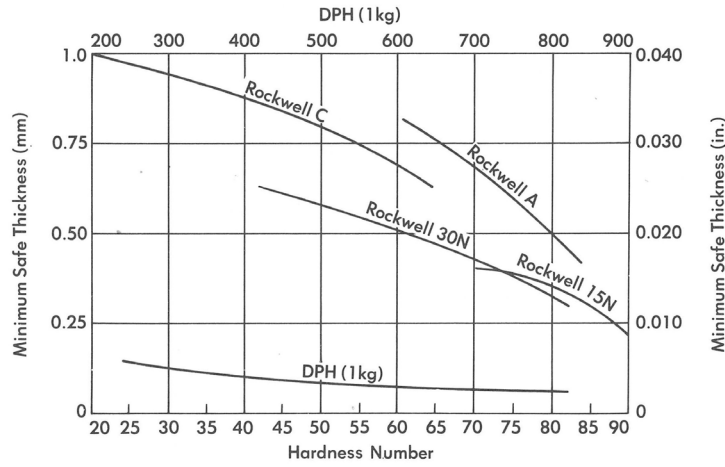
**Table 3–12. Typical High-Carbon Strip Thickness Tolerances.**

Thickness: mm (in.)	Thickness Tolerance: ± mm (in.)	
	Strip Width: mm (in.)	
	12.7–76.1 (0.50–2.99)	76.2–304.8 (3.00–12.00)
0.10–0.25 (0.004–0.010)	0.005 (0.00020)	0.006 (0.00025)
0.25–0.51 (0.010–0.020)	0.006 (0.00025)	0.009 (0.00035)
0.51–0.76 (0.020–0.030)	0.009 (0.00035)	0.013 (0.00050)
0.76–1.02 (0.030–0.040)	0.010 (0.00040)	0.013 (0.00050)
1.02–1.52 (0.040–0.060)	0.013 (0.00050)	0.019 (0.00075)
1.52–2.03 (0.060–0.080)	0.025 (0.00100)	0.038 (0.00150)
2.03–2.54 (0.080–0.100)	0.038 (0.00150)	0.051 (0.00200)
2.54–3.18 (0.100–0.125)	0.051 (0.00200)	0.063 (0.00250)

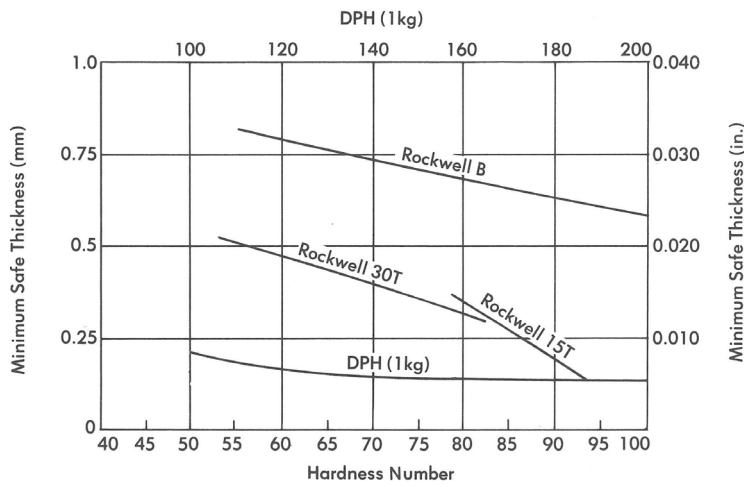
Precision rolled high-carbon steel strip is available commercially at tolerances considerably less than the values stated above.



**Fig. 3-8. Minimum Safe Thicknesses for Hardness Testing Hard Materials.**



**Fig. 3-9. Minimum Safe Thicknesses for Hardness Testing Soft Materials.**



*Table 3-13. Typical Properties of Spring Temper Alloy Strip.*

Material	Tensile Strength MPa (10 <sup>3</sup> psi)	Rockwell Hardness	Elongation(1) Percent	Bend Factor (1) (2r/t trans. bends)	Modulus of Elasticity 10 <sup>4</sup> MPa (10 <sup>6</sup> psi)	Poisson's Ratio
Steel, spring temper	1700 (246)	C50	2	5	20.7 (30)	0.30
Stainless 301	1300 (189)	C40	8	3	19.3 (28)	0.31
Stainless 302	1300 (189)	C40	5	4	19.3 (28)	0.31
Monel 400	690 (100)	B95	2	5	17.9 (26)	0.32
Monel K500	1200 (174)	C34	40	5	17.9 (26)	0.29
Inconel 600	1040 (151)	C30	2	2	21.4 (31)	0.29
Inconel X-750	1050 (152)	C35	20	3	21.4 (31)	0.29
Copper-Beryllium	1300 (189)	C40	2	5	12.8 (18.5)	0.33
Ni- Span-C	1400 (203)	C42	6	2	18.6 (27)	—
Brass CA 260	620 (90)	B90	3	3	11 (16)	0.33
Phosphor Bronze	690 (100)	B90	3	2.5	10.3 (15)	0.20
17-7 PH RH950	1450 (210)	C44	6	flat	20.3 (29.5)	0.34
17-7 PH Condition C	1650 (239)	C46	1	2.5	20.3 (29.5)	0.34

(1) Before heat treatment.

*Table 3-14. Recommended Hardness Scales for Hard and Soft Spring Alloys.*

Thickness: mm (in.)	Tempered Steel	Annealed Steel and Nonferrous Alloys
0.89 (0.035) and over	C	B
0.64-0.86 (0.025-0.034)	A	45T
0.35-0.61 (0.015-0.024)	30N	30T
0.20-0.36 (0.008-0.014)	15N	15T
Under 0.20 (0.008)	DPH	DPH

## 2 RESIDUAL STRESS, FATIGUE, AND RELIABILITY

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### 2.1 Introduction

One of the most difficult judgements required of a designer involves selection of operating stress level. Choice of operating stress level impacts the cost, performance, life, and reliability of the spring. Specific recommendations for operating stress levels are made in each section for each type of spring configuration. Performance of a spring is based on the resultant stress level experienced by the spring, which is an interaction of the applied and residual stress. Springmakers frequently induce residual stresses in springs to enhance performance. These stresses, while beneficial in one mode of spring operation, can be detrimental in another. Because recommendations on operating stress levels assume a particular set of residual stresses, it is important for the designer to have a basic knowledge of residual stresses. In this section, a general discussion on residual stress in springs, fatigue, and reliability is presented to assist the designer in selecting the operating stress level.

### 2.2 Residual Stresses

Residual stresses are the result of nonuniform plastic strains within a body. In an unrestrained body, residual stresses are balanced. That is, compressively stressed zones are balanced by zones stressed in tension. Residual stresses can be introduced by mechanical methods such as bending or twisting or by thermal methods such as quenching. Shot-peening is a special mechanical method designed to introduce residual compressive stresses at the surface. In the spring industry, favorable residual stresses are employed to increase load-carrying ability and/or increase fatigue strength.

### 2.3 Residual Stresses for Increased Load-Carrying Ability

As in the case of pre-stressed building structures, the goal in spring structures is to establish a residual stress opposite that encountered in the application. Highest operating stresses in springs are at the surface. For example, on a compression spring the objective is to establish residual compression at the surface, balanced by residual tension at the core in order to increase the load-carrying ability.

Some torsion springs, flat coil springs, and retaining rings have the desired residual stress pattern as a natural result of the forming process. The designer is cautioned that although residual stresses add to the load-carrying ability in one direction, these same residual stresses subtract from the load-carrying ability when the direction is reversed. For example, torsion springs have residual stress patterns allowing high load-carrying ability when the loading results in a reduction of the body coil diameter. Severe reductions in capacity and excessive set will result if torsion springs that have not been stress-relieved are loaded so as to increase the coil diameter.

Most springs do not contain beneficial residual stresses as a result of forming. After these springs have been stress-relieved, springmakers frequently induce a favorable residual stress pattern.

Regardless of what method is used, the favorable pattern is established by plastically deforming the spring in the same direction or in the same mode in which the ultimate application will deform the spring elastically. Unless plastic deformation occurs, no residual

stress, beneficial or otherwise, will be induced.

Knowing the necessary pre-deformation shape and/or dimensions that will lead to the desired finished shape is one aspect of the art and science of springmaking.

Compression springs are a good example of residual stress engineering. As coiled, material at the inner surface is in residual (bending with some twisting) tension; the outer surface is in residual compression. Neither is beneficial, so stress relieving is required. If after stress relief a spring does not set when compressed to "solid," it will not be residually stressed and will have a low ESC. If the stress in the spring exceeds the yield strength of the material when the spring is compressed to solid, plastic deformation will impart beneficial residual stress in the spring. Springmakers refer to this process as "removing set." Removing set can increase the load-carrying ability of springs by 45% to 65% and allows storage of twice the energy per pound of material.

Why aren't all springs made this way? For small springs, the procedures involve extra operations that may not be cost-effective. Some designs and applications are not amenable to set removal. For example, steel springs that require plating as a final operation should be free of residual stresses when plated. Finally, some springs, such as extension springs with high initial tension, cannot have set removed without removing an appreciable amount of initial tension.

## 2.4 Residual Stresses for Increased Fatigue Strength

In some applications, springs must operate for hundreds of millions of cycles. All materials have finite limits to their ability to withstand cyclic

stress. Residual stresses can either detract from the basic fatigue resistance of a material or enhance it. Many designers are accustomed to adding residual stresses to mean stress in order to estimate fatigue life. Residual stresses due to set removal, which increase the load-carrying ability of springs stressed in torsion, have only a small effect on fatigue performance and in most applications should be ignored. To simplify this handbook, recommended operating stress and methods used to estimate fatigue life have already taken into account residual stresses imparted to the spring by the springmaker. The designer's calculation should not make allowances for these residual stresses unless the residual stress pattern is likely to change prior to or during usage.

Shot-peening is a very effective means of enhancing fatigue performance of springs. A machine is used to propel steel shot with diameters ranging from 0.2 to 1.4 mm (0.008" to 0.055") at the spring. Propelled at high velocities, the shot particles impact the spring surface with enough energy to induce plastic deformation. This produces a shell on the outside portion of the peened part, which is biaxially stressed in compression. The balancing residual tension is located within the spring, well below the surface. These biaxial, residual compressive stresses lower the mean tensile stress on the surface of springs during cycling and significantly increase fatigue performance.

Residual peening stresses also minimize the detrimental effect of stress concentrations due to pits, scratches, seams, and other surface defects. Shot-peening does not heal defects in the material, but it does render any defects less likely to induce failure under a given set of operating conditions.

Shot-peening does not enhance load-carrying capacity of springs. In fact, peening can damage

this capacity by depressing yield strength of material in the plastically deformed zones. In springs with heavy sections, the peened zone represents a small fraction of total volume, so the effect is often undetectable. Fortunately, the set point of peened high-carbon steel springs can be restored by a low temperature heat treatment in the 200° to 250°C (392° to 482°F) range while preserving most of the benefits of the peening process. Exposure of peened parts to higher temperatures results in a significant loss of benefits. Shot-peening is detrimental to stress relaxation resistance, even at temperatures below 230°C (446°F) (Figures 3-1 and 3-2).

## 2.5 Fatigue

Fatigue is a vital element of spring engineering because materials have finite fatigue limits. Springs operate at high stresses, and springs frequently are required to operate for many cycles. Fatigue is an irreversible process that proceeds in three stages: crack initiation, crack propagation, and fracture. In order for fatigue to occur, the spring must be subjected to cyclic stresses, the stresses must have a tensile component, and there must be plastic strain. The plastic strain is often localized and too small to measure.

Fatigue is often considered as either high cycle (that which occurs under the influence of relatively low stress exposures and resulting in cycle life of greater than  $5 \times 10^5$  cycles) or low cycle (resulting from relatively high stress exposure leading to cycle life of less than  $5 \times 10^4$  cycles).

Fig. 4-1. Stress Definitions Used in Fatigue.

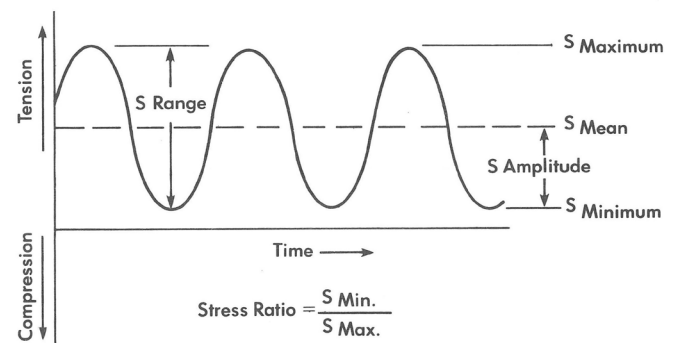
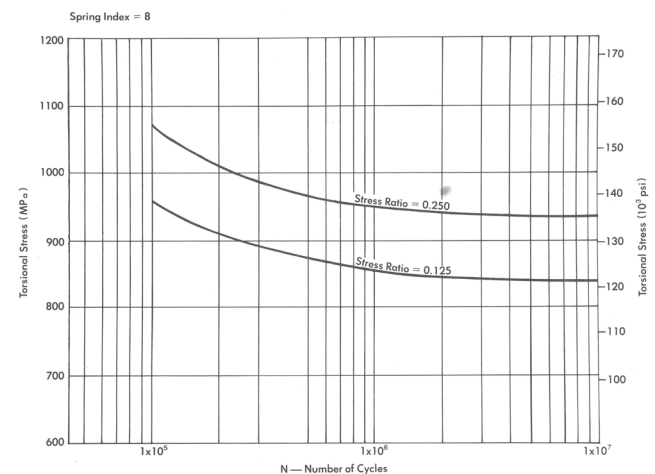


Fig. 4-2. S-N Curve for Helical Compression Springs. Springs Made from Music Wire ASTM A228, 1.3 mm (0.051") Diameter, Percent Survival: 90%, Material Lot No. 1321.



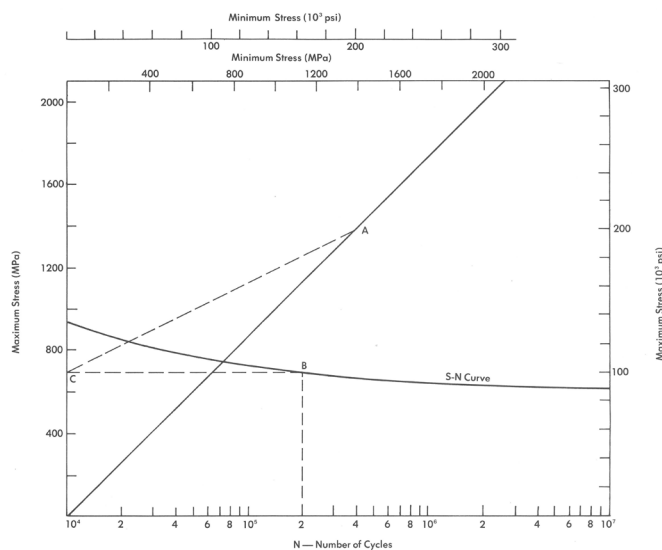
Precise boundaries for these regions have not been established and a gap exists between them. Relatively ductile spring materials perform well in low-cycle fatigue applications. Low-cycle fatigue life is not significantly improved by shot-peening nor severely reduced by minor surface imperfections. High-strength spring material performs well in high-cycle fatigue applications. High-cycle fatigue strength is markedly improved by shot-peening and severely reduced by seemingly minor surface imperfections.

Before presenting fatigue data, it is necessary to define some commonly used terms. Stress range ( $S_R$ ) is equal to the maximum stress ( $S_{max}$ ) minus the minimum stress ( $S_{min}$ ). Mean stress ( $S_{mean}$ ) is half the sum of maximum and minimum stresses. Stress amplitude ( $S_{amplitude}$ ) is equal to maximum

stress minus mean stress. These relationships are illustrated schematically in Figure 4-1. Stress ratio is the ratio of minimum stress to maximum stress. For most spring applications, the stress ratio is positive and in the zero to 0.8 range.

Typically, fatigue data are presented in the form of S-N curves (Figure 4-2), which is a graph of stress versus the log of the cycle life at a specified percent survival. Although it is common practice to specify a percent survival on S-N curves developed for specific applications, it is not possible to specify percent survival on S-N curves used for general design purposes, due to wide variations in applications, material quality, and springmaking practices encountered. There is a separate S-N curve for each material by size and also by stress type (torsion or bending). To predict part life at a different stress ratio, a Goodman diagram (Figure 4-3) can be combined with an S-N curve for the spring material in question.

Fig. 4-3. Estimating Fatigue Life from a Combined S-N Curve and Modified Goodman Diagram.



## 2.6 Fatigue Life Prediction

First, an S-N curve is developed from the test data at a known stress ratio, and corrected to

a 0-stress ratio using a modified Goodman method. In a typical modified Goodman diagram (4-3), the abscissa is used as a double scale—a log scale for the number of cycles,  $N$ , and a linear scale labeled “minimum stress” with the same scale as the ordinate or maximum stress scale.

Second, a  $45^\circ$  line is drawn from the origin of the plot. On this line, point A is marked corresponding to the ultimate strength of the material. This is the tensile strength for structures in bending and torsional strength for structures in torsion. Torsional strength can be estimated as two-thirds of tensile strength. These two lines constitute the combined S-N and Goodman diagrams for the given material and state of stress.

Then to develop a specified service life, a vertical line is drawn from the appropriate value on the  $N$  scale to the S-N curve. At the intersection B, a horizontal line is drawn to intersect the ordinate or maximum stress scale. From point C, a straight line is drawn to the tensile strength point A. Along line AC lie the stress combinations that meet the desired life.

This procedure for estimating fatigue life is approximate and should only be used in the absence of specific data. For critical applications, it is necessary to test prototype springs to obtain data under conditions that simulate operation.

## 2.7 Reliability

Reliability is the probability that a spring will perform for a given period of time at a specified confidence level. Because reliability of spring components is often critical to system performance, it is important that the designer consider the reliability required, either quantitatively or qualitatively, and inform the

springmaker. The most common failure modes for springs are fracture due to fatigue and excessive loss of load due to stress relaxation. Different methods are required to predict reliability for each failure mode. A simple approach to reliability is presented below. The interested reader is referred to treatments in References 1 and 2.

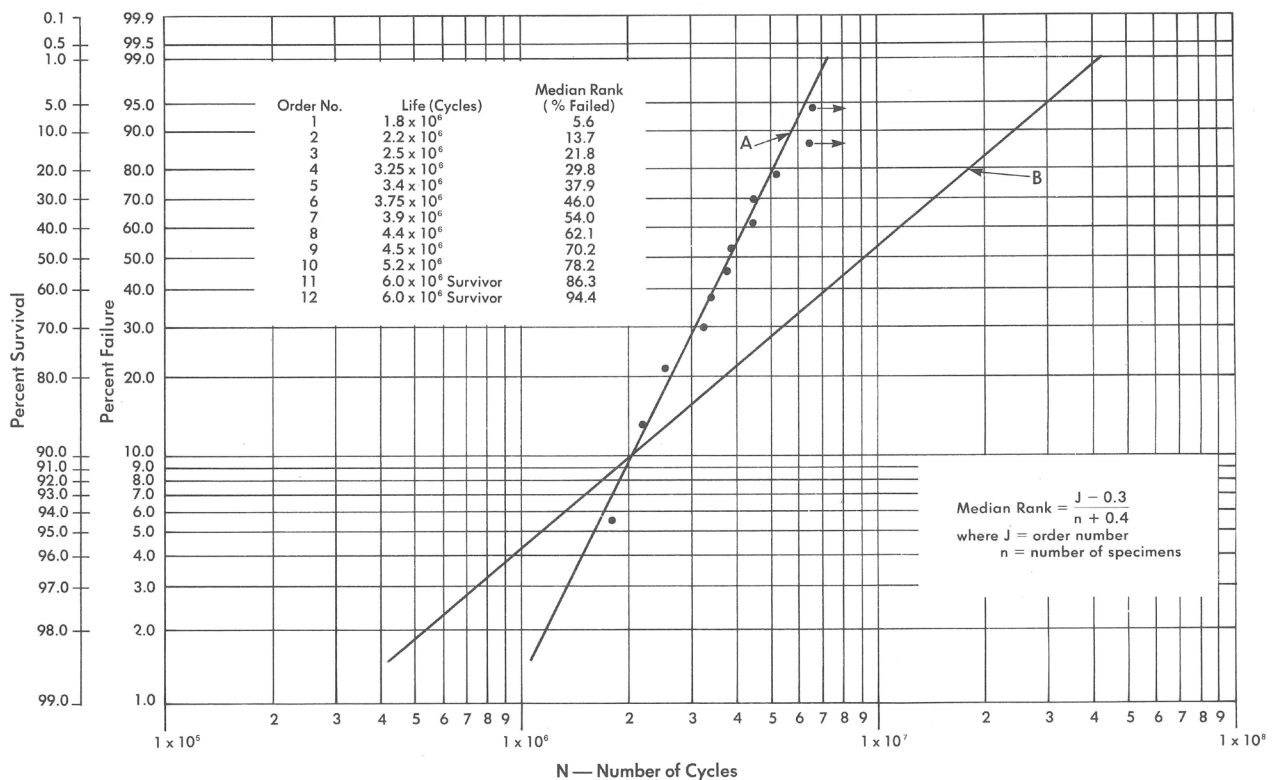
### 2.8 Reliability and Fatigue

Many variables—such as operating stress, operating environment, material strength, and surface condition—interact to cause a range of lives. Distribution of fatigue life is not normal or Gaussian. The Weibull distribution provides a useful means to analyze fatigue life data. To predict the percent survival, test results are first ranked in order of ascending life.

A median rank is assigned to each failure on the basis of number of specimens tested. Median rank is the order number minus 0.3 divided by number of specimens plus 0.4. Order number is the rank within sample, and sample number is the total number of specimens. If the sample size is 12, median rank for the second failure is 13.7%.

Median ranks are plotted against life using a computer program (Figure 4-4). If the plot of test failures follows a straight line, the line may be extrapolated to estimate the percentage of the population that is expected to survive a specific number of cycles. Accuracy of the extrapolation increases as the number of failures increases. For example, based on line A in Figure 4-4, 90% of the population is expected to survive 2 million cycles. The statistical “confidence” for line A is 50%. This simply states that if a large number of identically selected groups of As are tested,

Fig. 4-4. Weibull Method for Estimating Reliability.



results are expected to equally distribute on either side of line A. The percent survival is now defined in Figure 4-4 and is expressed as 90% of the springs will survive 2 million cycles with confidence level of 50%. Life of 2 million cycles is sometimes referred to as the  $S_{10}$  life. The term  $S_{10}$  applies to the number of cycles where 10% of the springs will fail and 90% will survive.

Specifying a high survival rate of 99.9% with a high level of confidence (95% to 99%) requires extensive testing, which is not warranted in most situations. To assure high reliability levels and avoid excessive testing costs, designers often specify  $S_{10}$  life test conditions that are considerably more severe than encountered in the application.  $S_{10}$  life is often used as a quality assurance acceptance criterion.

Although  $S_{10}$  life is a good measure of reliability, springs with the same  $S_{10}$  life can have different reliabilities at other survival rates, as illustrated by curve B in Figure 4-4. Curve B has a lower slope than curve A, indicative of more scatter in the data. Even though the  $S_{10}$  lives are identical, the first failure from B population will occur before the first failure from A population. Conversely, the last failure from B population will occur after the last failure from A population. Variations in

surface quality and test environments contribute to the scatter or variations in slope of the Weibull curves.

## 2.9 Reliability and Load Loss

Load loss due to stress relaxation is usually controlled by specifications that require the spring to be held in a deflected position at a test temperature for a period of time. Relaxation is then expressed as a percent of load loss at a test height by the following formula:

$$\frac{P_o - P_f}{P_o} \times 100 = \% \text{ load loss}$$

$P_o$  is the load at test height before stress relaxation testing, and  $P_f$  is the load at test height after testing. When large numbers of springs are tested, the observed percent of load loss is normally distributed. Load loss requirements are best specified as maximum average load loss (50% confidence) or maximum load loss at another confidence level such as 97.5%. Use of very high confidence levels requires extensive testing. For designs that require very high reliability, the test time, temperature, and/or stress should be considerably more severe than conditions encountered in operation.

## 3 HELICAL COMPRESSION SPRINGS

### 3.1 Introduction

Helical compression springs are used to resist applied compression forces or to store energy in the push mode. They have the most common spring configuration and are found in many applications such as automotive, aerospace, and consumer goods. While the most prevalent form of compression spring is a straight cylindrical spring made from round wire, many other forms are produced. Conical, barrel, hourglass, or cylindrical forms are available, with or without variable spacing between coils. Such configurations are used to reduce solid height, buckling, and surging, or to produce nonlinear load deflection characteristics. ESC is greater for round wire compression springs than for rectangular wire compression springs and can be increased by nesting. Rectangular wire is sometimes employed to reduce solid height or increase the space efficiency of the design. Most die springs are made from rectangular wire for this reason. The Stock Precision Engineered Components (SPEC) 0029ccbv line of springs contains hundreds of compression spring designs using wire sizes from 0.15 to 5.26 mm (0.006" to .207") diameter music or stainless-steel wire. Specifying SPEC springs saves design time, reduces cost for low volume applications, and offers improved delivery.

### 3.2 Helical Compression Spring Terminology

Special terminology has evolved in the spring industry to describe features of helical compression springs. These terms are defined and the relationship between terms is reviewed in Figure 5-1. Communication between designer

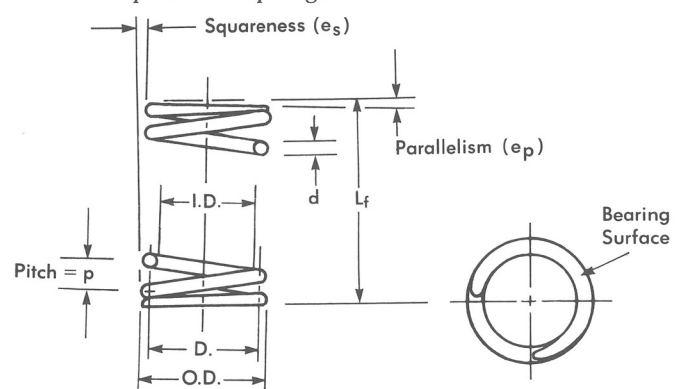
and springmaker is improved if these common terms are used.

### 3.3 Spring Diameter

Outside diameter (O.D.), inside diameter (I.D.), and mean diameter are all used to describe helical compression spring dimensions. Mean diameter is equal to the sum of O.D. and I.D. divided by two, and is employed in spring design calculations for stress and deflection. The O.D. is specified for springs that operate in a cavity, while the I.D. is specified for springs that operate over a rod, seat, or shaft. Minimum diametral clearance between the spring and cavity or rod is as follows:

- 0.05D when  $D_c$  is greater than 13 mm (0.512")
- 0.10D when  $D_c$  is less than 13 mm (0.512")
- $D_c$  is the diameter of the rod or cavity

*Fig. 5-1. Dimensional Terminology for Helical Compression Springs.*



Diameter increases when a spring is compressed. Although the increase in diameter is usually small, it must be considered when minimum clearances are established. The increase in diameter is a function of initial spring pitch and

can be estimated from the following equation where  $p$  = pitch.

$$O.D. \text{ at solid} = \sqrt{D^2 + \frac{p^2 - d^2}{\pi^2}} + d$$

If the spring ends are allowed to unwind, the O.D. at solid may be greater than calculated by this equation. Long springs buckle (see Figure 5-8) and may require lateral support and larger diametral clearances.

### 3.4 Spring Index

Spring index is the ratio of mean diameter to wire diameter or radial dimension of the cross-section (Figure 5-15). The preferred index range is 4 to 12. Springs with high indexes tangle and may require individual packaging, especially if the ends are not squared. Springs with indexes lower than 4 are difficult to form.

### 3.5 Free Length

Free length is overall spring length in the free or unloaded position (Figure 5-1). If loads are not critical, free length should be specified. When definite loads are required, free length could be a reference dimension that can be varied to meet load requirements. Pitch is the distance between centers of adjacent coils and is related to free length and number of coils.

### 3.6 Type of Ends

Types of ends available are plain ends, plain ends—ground, squared ends, and squared ends—ground (Figure 5-2). To improve squareness and reduce buckling during operation, a bearing surface of at least 270° is required. Squared and ground springs are normally supplied with a bearing surface of

270° to 330°. Additional grinding results in thin sections. “Squared ends only” are preferred on springs with small wire diameters (less than 0.5 mm or 0.020”), a large index (greater than 12), or low spring rates. Squared ends cost less to manufacture than squared and ground ends.

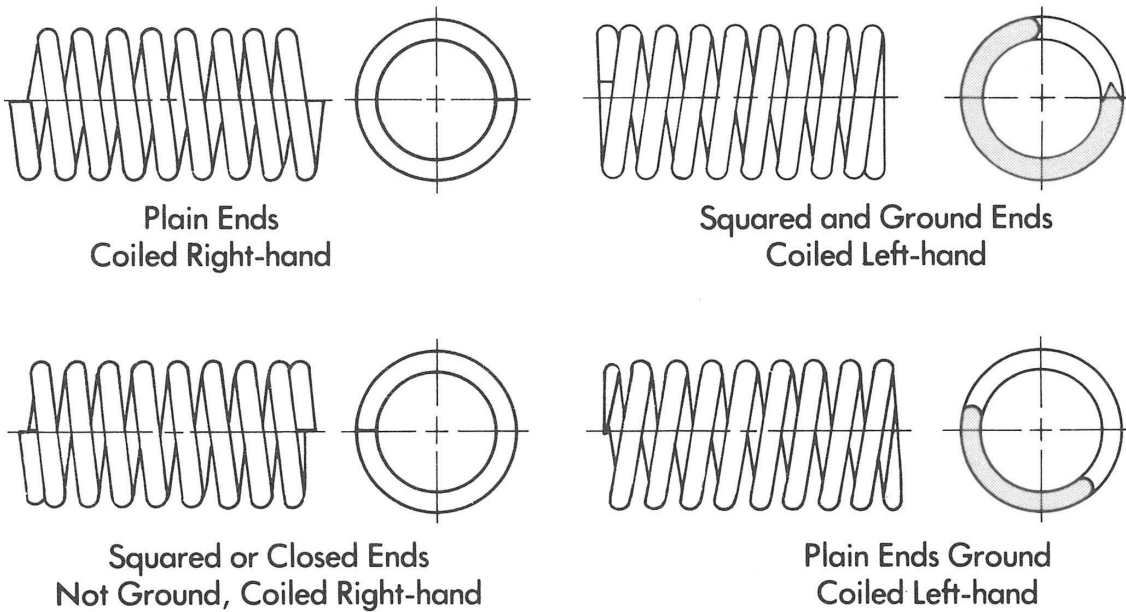
### 3.7 Number of Coils

Total number of coils should be specified as a reference number. For springs with squared ends, the total number of coils minus two is the number of active coils. There is some activity in end coils, but during deflection some active material comes in contact with the end coils and becomes inactive. Experience indicates that this equation is a good approximation. The number of active coils in springs with plain ends is greater than those with squared ends, and depends upon the seating method employed. Some useful guidelines for estimating the number of active coils are presented in Table 5-1.

### 3.8 Solid Height

Solid height is the length of a spring with all coils closed. For ground springs, solid height is the number of coils multiplied by wire diameter. For unground springs, solid height is the number of coils plus one, multiplied by wire diameter (Table 5-1). If critical, solid height should be specified as a maximum dimension. After allowances are made for plating or other coatings, it is good practice to add one-half of the wire diameter to determine maximum solid height. With larger wire sizes and fewer coils, this allowance can be decreased. Solid height is often measured by applying a force equal to 110% to 150% of the calculated load at solid. If solid height is not critical, this dimension should be omitted.

**Fig. 5-2. Types of Ends for Helical Compression Springs.**



**Table 5-1. Guidelines for Dimensional Characteristics of Compression Springs.**

Dimensional Characteristics	Type of Ends			
	Open or Plain (Not ground)	Open or Plain (Ground)	Squared Only	Squared and Ground
Solid Height ( $L_s$ )	$(N_t + 1)d$	$N_t d$	$(N_t + 1)d$	$N_t d^*$
Pitch ( $p$ )	$\frac{L_f - d}{N_a}$	$\frac{L_f}{N_t}$	$\frac{L_f - 3d}{N_a}$	$\frac{L_f - 2d}{N_a}$
Active Coils ( $N_a$ )	$\frac{L_f - d}{p}$	$\frac{L_f}{p} - 1$	$\frac{L_f - 3d}{p}$	$\frac{L_f - 2d}{p}$
Total Coils ( $N_t$ )	$N_a$	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free Length ( $L_f$ )	$p N_t + d$	$p N_t$	$p N_a + 3d$	$p N_a + 2d$

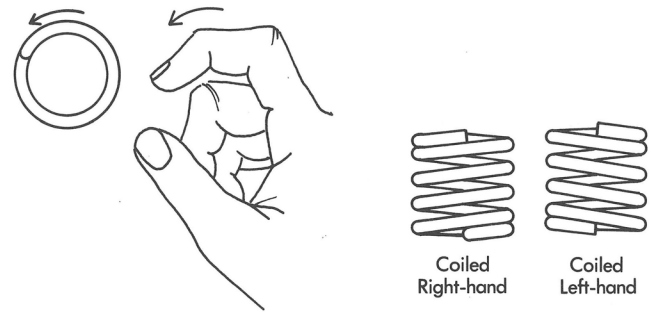
\*For small index springs lower solid heights are possible.

### 3.9 Direction of Coiling

A helical compression spring can be either left- or right-hand coiled. If the index finger of the right hand can be bent to simulate direction of coil, so that the fingernail and coil tip are approximately at the same angular position, the spring is right-hand wound (Figure 5-3). If the index finger of the left hand simulates the coil direction, the spring is left-hand wound. If direction of coiling is not specified, springs may be coiled in either direction. Nested springs with small diametral clearances should be coiled in opposite directions.

support system due to a tendency of the ends to rotate as the spring is compressed. Hysteresis for compression springs is low, and the contribution due to internal friction in the spring material itself is generally negligible.

*Fig. 5-3. Direction of Coiling Helical Compression Springs.*



### 3.10 Squareness and Parallelism

Squareness of helical compression springs can be measured by standing a sample spring on end on a horizontal flat plate and bringing the spring against a straightedge at right angles to the plate. The spring is rotated to produce a maximum out-of-square dimension  $e_s$  (Figure 5-1). Normally squared and ground springs are square within  $3^\circ$  when measured in the free position. Squareness should be checked at both ends. Specifying squareness or parallelism in the free position does not assure squareness or parallelism under load.

Parallelism (Figure 5-1) refers to the relationship of the ground ends, and is determined by placing a spring on a flat plate and measuring the maximum difference in free length around the spring circumference  $e_p$ .

### 3.11 Hysteresis

Hysteresis is the loss of mechanical energy under cyclic loading and unloading of a spring. It results from frictional losses in the spring

### 3.12 Spring Rate

Spring rate for helical compression springs is defined as the change in load per unit deflection and is expressed as shown:

$$k = \frac{P}{f} = \frac{Gd^4}{8D^3N_a}$$

This equation is valid when the pitch angle is less than  $15^\circ$  or deflection per turn is less than  $D/4$ . For large deflections per turn, a deflection correction factor (Reference 3) should be employed.

The load deflection curve for helical compression springs is essentially a straight line up to the elastic limit, provided that the amount of active material is constant.

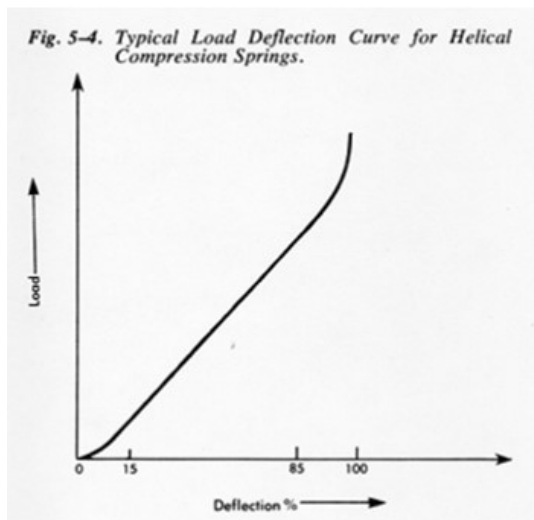
The initial spring rate and the rate as the spring approaches solid often deviate from the average calculated rate. When it is necessary to specify a rate, it should be specified between two test heights that lie within 15% to 85% of the full deflection range (Figure 5-4).

When compression springs are used in parallel,

the composite rate is the sum of the rates for individual springs. For compression springs in series, the rate is calculated from the following:

$$k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \dots \frac{1}{k_n}}$$

This relationship is often used to calculate the rate for springs with variable diameters. The technique involves dividing the spring into many small increments and calculating the rate for each increment. The rate for the whole spring is computed from the rate of the increments according to the equation above.



### 3.13 Stress

Wire in a helical compression spring is stressed in torsion. Torsional stress is expressed as:

$$S = \frac{8PD}{\pi d^3} K_W$$

Bending stresses are present but can be ignored except when the pitch angle is greater than 15° and deflection of each coil greater than D/4 (Reference 3). Under elastic conditions, torsional stress is not uniform around the wire cross-section due to coil curvature and a direct shear load. Maximum stress occurs at the inner surfaces of the spring and is computed using a stress correction factor. The most widely used stress correction factor  $K_{W1}$  is attributed to Wahl. It is shown below and in Figure 5-5.

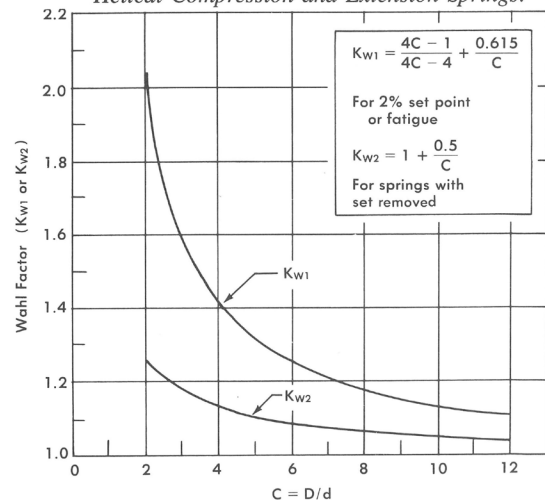
$$K_{W1} = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

In some circumstances after yielding occurs, resultant stresses are distributed more uniformly around the cross-section. Then, a stress correction factor  $K_{W2}$  (which accounts only for the direct shear) component is used.

$$K_{W2} = 1 + \frac{0.5}{C}$$

In other circumstances, such as static loading at elevated temperatures, stress distribution tends to become uniform around the cross-section and can best be estimated by using no correction factor. Use of different stress correction factors can lead to confusion. In published data, it is essential to know which stress correction factors were used. (The stress correction factor used by a designer must be the same as that used to develop the data.) Methods to calculate stress for different applications and the use of stress correction factors will be discussed in the following paragraphs on choice of operating stresses.

Fig. 5-5. Wahl Stress Correction Factors for Round Wire Helical Compression and Extension Springs.



### 3.14 Loads

When deflection is known, loads are determined by multiplying deflection by the spring rate. When the stress is known or assumed, loads are



determined from Equation 5-4. The procedure used to determine loads of variable rate springs is complex. In this case, the load deflection curve is approximated by a series of short chords. The spring rate is calculated for each chord and multiplied by deflection to obtain the load. The load is then added to that calculated for the next chord. The process is repeated until load has been calculated for the desired value of deflection (Figure 5-6).

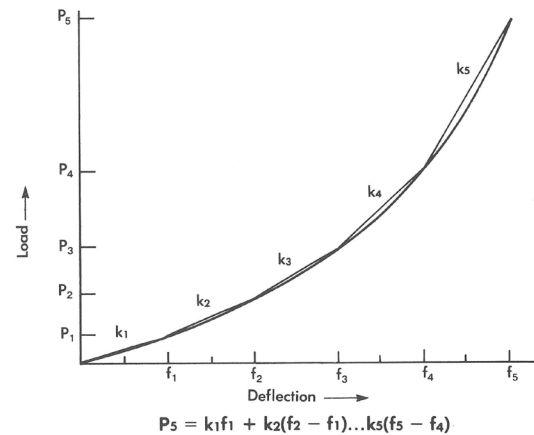
Loads should be specified at a test height. Because the load deflection curve is often not linear at very low loads or at loads near solid, loads should be specified at test heights between 15% and 85% of the full deflection range (Figure 5-4).

Loads are classified as static, cyclic, or dynamic. In static loading applications, the spring is expected to operate between specified loads only a few times. Frequently, springs in static applications remain loaded for long periods of time. In typical cyclic applications, springs are required to cycle between specified loads from 10,000 to more than a billion cycles. During dynamic loading, the rate of load application is high and causes a surge wave in the spring that will induce stresses that exceed the value calculated from Equation 5-4.

### 3.15 Buckling of Compression Springs

Compression springs that have lengths greater than four times the spring diameter can buckle. If properly guided, either in a tube or over a rod, buckling can be minimized. However, friction between the spring and tube or rod will affect the loads, especially when the aspect ratio ( $Lf/D$ ) is high.

Fig. 5-6. Load Deflection Curve for a Variable Rate Spring.



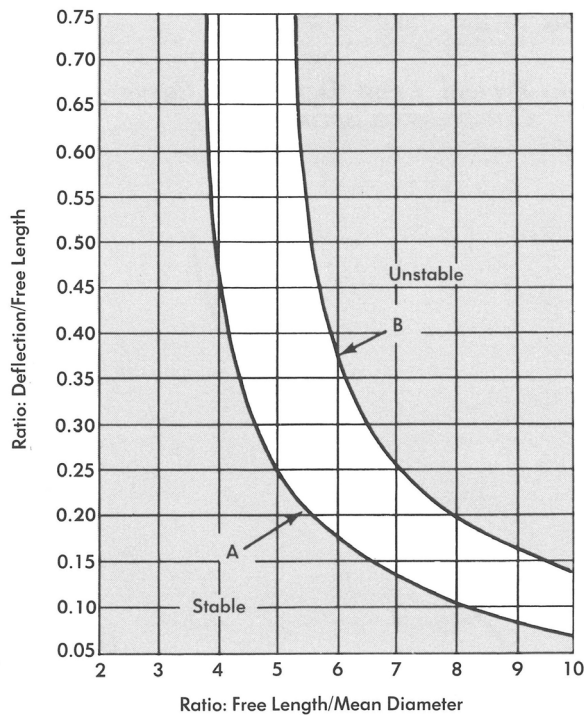
Critical buckling conditions are shown in Figure 5-7 for axially loaded springs with squared and ground ends. Curve A is for springs with one end on a flat plate and the other end free to tip (Figure 5-8). It indicates that buckling will occur when the spring design is above and to the right of the curve. A tendency for buckling is considerably less for springs compressed between parallel plates as shown in curve B. For applications requiring springs with a high aspect ratio and large deflections, several springs can be used in series in a tube or over a rod, with guides between the springs to prevent binding.

### 3.16 Choice of Operating Stress—Static Conditions

For static applications, the yield strength or stress relaxation resistance of the material limits the load-carrying ability of a spring. The spring is required to operate for a limited number of cycles, and the velocity of the end coils is low to preclude high stresses due to surging or impact conditions. Maximum allowable torsional stress for helical compression springs used in static applications is presented in Table 5-2 as a percentage of the tensile strength for common spring materials. For springs that do not contain beneficial residual stresses induced by set removal, maximum allowable torsional

stress values are from 35% to 50% of the tensile strength. To calculate the stress before set removal, it is necessary to use the  $K_{W1}$  correction factor. If the calculated stress at solid is greater than the indicated percentage of tensile strength, the spring will take a permanent set when deflected to solid. Amount of set is a function of the amount that calculated stress at solid exceeds the indicated percent of tensile strength.

Fig. 5-7. Critical Buckling Condition Curves.



To increase the load-carrying ability of springs in static applications, it is common practice to make the spring longer than its required free length and to compress the spring to solid. This causes the spring to set to the desired final length and induces favorable residual stresses. This process is called "removing set" or pre-setting and can be conducted at either room or elevated temperatures. The loss of deflection from the free position to solid by cold set removal should be at least 10%. If the set is less, it is difficult to control the spring's free length. Ratios of stress

greater than 1.3 lead to distortion and do not appreciably increase the load-carrying ability. This is illustrated schematically in Figure 5-9.

Allowable torsion stresses in springs with set removed (Table 5-2) are significantly higher than for springs that have not had set removed. It is important to note that because yielding has occurred during pre-setting, the stress is relatively uniform around the cross-section, and it is calculated using the  $K_{W2}$  stress correction factor.

Set removal is an added springmaking operation that increases the manufacturing cost but greatly increases the ESC of the spring. Set removal is common for critical springs made from premium materials. In some instances, springs have the set removed during an assembly operation.

Fig. 5-8. End Conditions Used to Determine Critical Buckling.

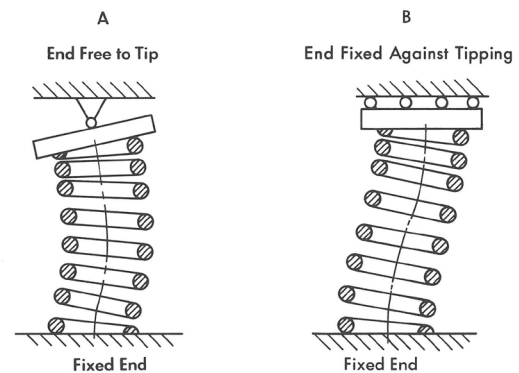


Table 5-2. Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications. Bending or buckling stresses not included.

Materials	Maximum % of Tensile Strength	
	Before Set Removed ( $K_{W1}$ )	After Set Removed ( $K_{W2}$ )
Patented and cold drawn carbon steel	45	60-70
Hardened and tempered carbon and low alloy steel	50	65-75
Austenitic stainless steels	35	55-65
Nonferrous alloys	35	55-65



If the calculated stress using the  $K_{W2}$  stress correction factor exceeds the percentage of tensile strength indicated in Table 5-2, the spring cannot be made. In this case, it is necessary to either lower the stress by altering spring design or selecting a higher strength material.

In some applications, maximum operating stress is limited by material stress relaxation resistance and amount of load loss that the design can tolerate. When load is constant, these designs are limited by material creep resistance. When the spring is compressed at a fixed test height, stress relaxation resistance of the material is limiting. Designs limited by stress relaxation resistance are more common than designs limited by creep resistance. It is suggested that creep-limited designs be reviewed by Associated Spring engineers.

Stress relaxation is defined as percent load loss according to the following relationship:

$$\% \text{ Relaxation} = \frac{P_O - P_F}{P_O} \times 100$$

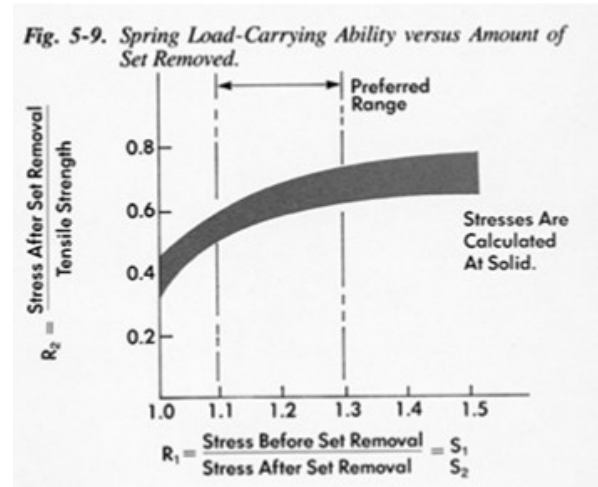
$P_O$  is load at test height before testing.

$P_F$  is load at test height after testing.

Typical stress relaxation data (Figure 5-10) indicate that at high stresses, some spring materials (such as music wire) exhibit appreciable stress relaxation after only 100 hours at temperatures as low as 100°C (212°F). These data are only representative of the conditions indicated. Stress relaxation is affected by material, spring processing variables, time, temperature, and stress. Associated Spring engineers should be contacted for critical applications involving stress relaxation resistance.

When set is removed at an elevated temperature, the process is called heat setting. It significantly improves the stress relaxation resistance of springs (Figure 3- 2) at moderate temperatures and is frequently a more cost-effective method

for achieving low levels of stress relaxation than specifying a more costly spring material.



### 3.17 Choice of Operating Stress—Cyclic Applications

In cyclic applications, the load-carrying ability of a spring is limited by material fatigue strength. Velocity of end coils is low compared to the natural frequency. To select the optimum stress level, it is necessary to balance spring cost versus reliability. Reducing operating stresses increases spring reliability as well as cost. A complete knowledge of operating environment, expected life, stress range, frequency of operation, speed of operation, and permissible levels of stress relaxation are required in order to make the best choice between cost and reliability.

Because maximum stress is at the wire surface, any surface defects such as pits or seams severely reduce fatigue life. Shot-peening improves fatigue life and minimizes the harmful effect of surface defects, but it does not totally remove them.

Maximum allowable design stresses for fatigue applications should be calculated using the  $K_{W1}$  correction factor and are shown for common spring materials in Table 5-3. These values are for a stress ratio of zero in an ambient environment with no surging. Note that shot-

peening increases the fatigue strength by as much as 20% at lives of 10 million cycles.

Values in Table 5-3 are guidelines for designers and should only be used in the absence of specific data. Most springs designed to recommended stress levels will exceed the indicated lives; however, in the absence of detailed information on material, manufacturing method, and operating conditions, it is not possible to quantify the reliability level.

### 3.18 Fatigue Life Estimation Example

Fatigue life at other stress ratios can be determined from Table 5-3 according to the procedures outlined in Section 4.

A short example illustrates the procedure:

Estimate the fatigue life of a not-shot-peened helical compression spring loaded sinusoidally at a rate of one cycle per second. The spring is flooded with oil and operates at a maximum temperature of 40°C (104°F). The material is ASTM A228 wire and ends are squared and ground. The design is given here:

- $d = 1.00 \text{ mm (0.039")}$
- $C = 8$
- $L_f = 20.5 \text{ mm (ref) (0.807")}$
- $L_1 = 17.5 \text{ mm (0.689")}$
- $L_2 = 10 \text{ mm (0.394")}$
- $L_s = 8 \text{ mm (0.315")}$
- $N_t = 8$

Spring rate is determined from equation:

$$k = \frac{Gd^4}{8D^3N_a} = 3.2 \text{ N/mm}$$

Loads are calculated from the deflections and found to be:

$$P_1 = (20.5 - 17.5) \times 3.2 = 9.6 \text{ N}$$

$$P_2 = (20.5 - 10.0) \times 3.2 = 33.6 \text{ N}$$

$$P_s = (20.5 - 8) \times 3.2 = 40 \text{ N}$$

Stresses are calculated using Equation 5-4 and are:

$$S = \frac{8PD}{\pi d^3} K_{W1}$$

$$S_1 = 232 \text{ MPa}$$

$$S_2 = 810 \text{ MPa}$$

$$S_s = 965 \text{ MPa}$$

Tensile strength of the wire is 2180 MPa (Figure 3-3). The stress at solid is 44% of the tensile strength. Referring to Table 5-2, the maximum stress allowable before set removal for ASTM A228 is 45% of tensile strength. Therefore, the spring can be made and does not require set removal.

To estimate the fatigue life, it is necessary to do the following:

1. Plot an S-N curve on a modified Goodman diagram (Figure 5-11) using the data from Table 5-3 for not-shot-peening springs and a tensile strength of 2180 MPa.
2. Plot point A on the 45° line at 67% of the tensile strength.
3. Plot the stress range coordinates, point B.
4. Estimate the life by drawing a line through AB. At the intersection of this line with the vertical axis, point C, draw a horizontal line to intersect a S-N curve. The point

of intersection,  $D$ , is the estimated life of 2,500,000 cycles.

Velocity of the surge wave  $V_s$  varies with material and spring design, but is usually in the range of 50 to 500 m/sec. The surge wave limits the rate at which a spring can absorb or release energy by limiting impact velocity  $V$ .

### 3.19 Dynamic Loading—Impact

When a spring is loaded or unloaded, a surge wave is established that transmits torsional stress from the point of loading along the spring length to the point of restraint. The surge wave travels at a velocity approximately one-tenth of the normal torsional stress wave. Velocity of the torsional stress wave ( $V_T$ ) is given by:

$$V_T = 10.1 \sqrt{\frac{Gg}{\rho}} \text{ m/sec (or) } V_T = \sqrt{\frac{Gg}{\rho}} \text{ in./sec} \quad (5-8)$$

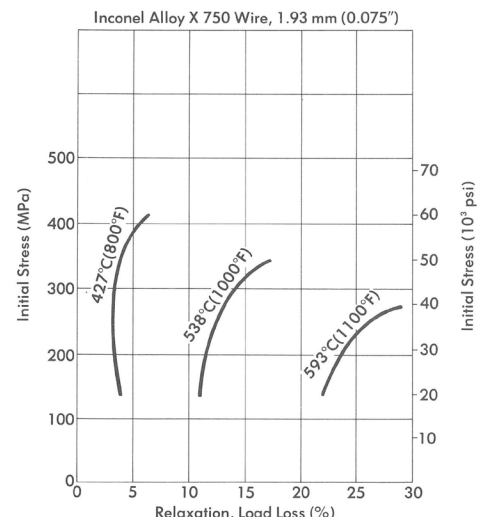
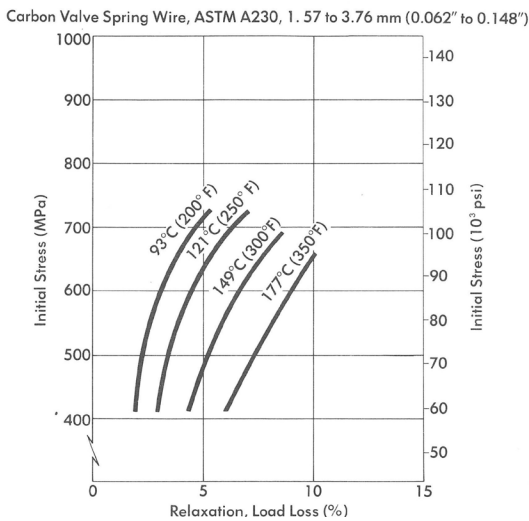
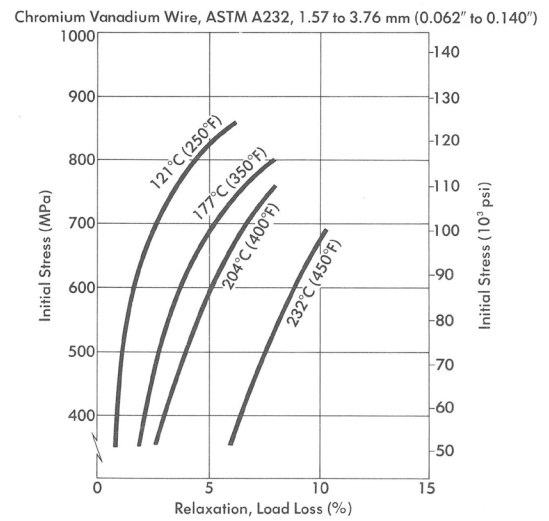
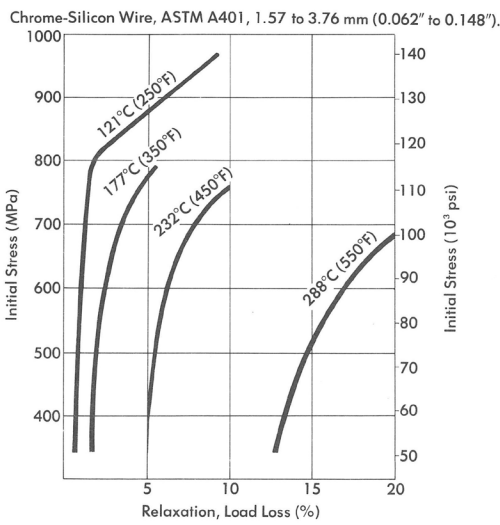
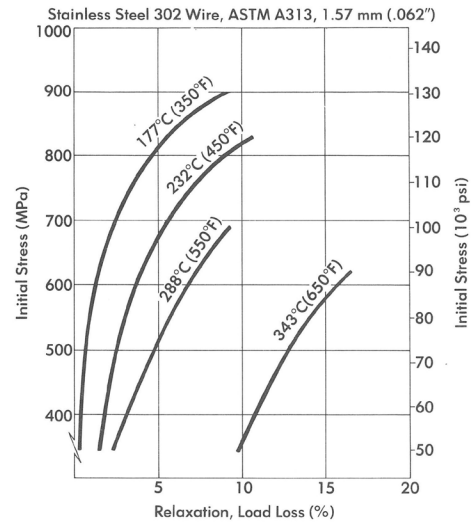
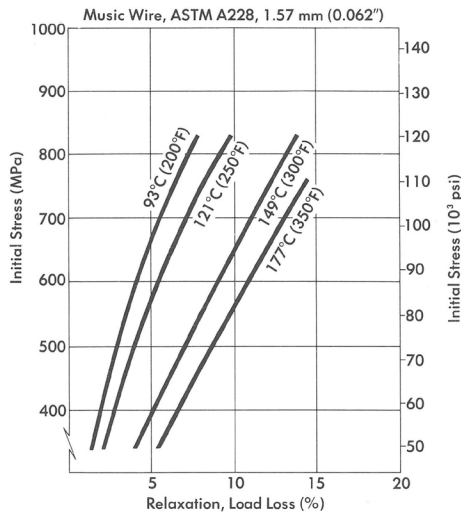
**Table 5-3. Maximum Allowable Torsional Stresses for Round Wire Helical Compression Springs in Cyclic Applications.**

Fatigue Life (cycles)	Percent of Tensile Strength			
	ASTM A228, Austenitic Stainless Steel and Nonferrous		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened	Not Shot-Peened	Shot-Peened
$10^5$	36	42	42	49
$10^6$	33	39	40	47
$10^7$	30	36	38	46

This information is based on the following conditions: no surging, room temperature and noncorrosive environment.

$$\text{Stress ratio in fatigue} = \frac{S_{\text{minimum}}}{S_{\text{maximum}}} = 0$$

**Fig. 5-10. Spring Relaxation Data for Various Materials.** Springs were preset at room temperature and tested 100 hours at the indicated temperatures. The initial stress is  $K_{W1}$  corrected.



Impact velocity is the spring velocity parallel to the spring axis and is a function of stress and material constants shown as:

$$V \approx 10.1S \sqrt{\frac{g}{2\rho G}} \text{ m/sec (or) } V \approx S \sqrt{\frac{g}{2\rho G}} \text{ in./sec}$$

This is a surprising result because impact velocity and stress are independent of the spring configuration. For steels, impact velocity reduces to:

$$V = \frac{S}{35.5} \text{ m/sec (or) } V = \frac{S}{131} \text{ in./sec}$$

If a spring is compressed to a given stress level and released instantaneously, the maximum spring velocity is expressed as the stress divided by 35.5. Similarly, if a spring is loaded at a known velocity, instantaneous stress can be calculated. At very high loading velocities, instantaneous stress will exceed the stress calculated from the conventional static formula (Equation 5-4) and will limit design performance. These equations for impact velocity are only concerned with the primary surge wave. Frequently, this wave will reflect from the other end of the spring, increasing stress. Springs loaded at high velocities are frequently subject to resonance phenomena.

When the ratio of the weight to be accelerated to the weight of the spring is less than 1, surge wave theory accurately predicts design performance (Figure 5-12). At high weight ratios and lower velocities, an energy balance is used to predict velocity of a weight projected from the spring end or deflection of the spring when impacted by a mass. Velocity and deflection are related as:

For horizontal loading:

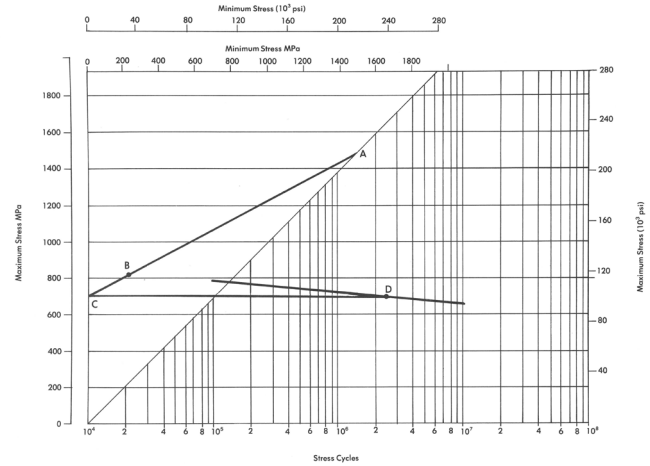
$$f = 31.6V \sqrt{\frac{W}{gk}} \text{ mm (or) } f = V \sqrt{\frac{W}{gk}} \text{ in.}$$

For vertical loading:

$$f = 31.6V \sqrt{\frac{W}{gk} + \frac{W}{k}} \text{ mm (or) } f = V \sqrt{\frac{W}{gk} + \frac{W}{k}} \text{ in.}$$

$W/g$  is the mass that is being accelerated or decelerated, and  $V$  is the axial velocity of the spring.

Fig. 5-11. Modified Goodman Diagram for Estimating Fatigue Life.



These equations assume that the spring is massless and should only be used when the spring mass is less than one-quarter of the mass to be accelerated.

When the ratio of spring load to weight is less than 4, the energy required to accelerated the spring itself becomes appreciable. By assuming that all mass of the spring is concentrated at the moving end, Equations 5-10 and 5-11 can be corrected by substituting  $(W + W_s/3)$  for  $W$  where  $W_s$  is the spring weight.

### 3.20 Dynamic Loading—Resonance

Resonance occurs in a spring when the frequency of the cyclic loading is near natural spring frequency or a multiple of it. Resonance can increase individual coil deflection and stress levels well above amounts predicted by static or equilibrium analysis. Resonance can also cause spring bounce, which results in loads considerably lower than calculated at the minimum spring deflection. To avoid resonance, natural spring frequency should be at least 13 times the operating frequency.



The natural frequency of a compression spring is inversely proportional to the time required for a surge wave to traverse the spring. For a compression spring without damping and with both ends fixed:

$$n = \frac{1.12 \times 10^3 d}{D^2 N_a} \sqrt{\frac{Gg}{\rho}}; \text{ for steel } n = \frac{3.5 \times 10^5 d}{D^2 N_a} \text{ metric}$$

$$n = \frac{d}{9D^2 N_a} \sqrt{\frac{Gg}{\rho}}; \text{ for steel } n = \frac{14000 d}{N_a D^2} \text{ English}$$

If a spring cannot be designed so the natural frequency is more than 13 times operating frequency, or if the spring is to serve as a vibration damping device, it must utilize one of several methods of energy absorption. Generally, these are friction devices in which the spring rubs against another element such as an internal damper coil, arbor, housing, or another portion of the spring. Variable pitch springs and springs in combination are also occasionally used to avoid or minimize resonant frequency effects.

For a vibration isolation system, the essential characteristic is that the natural frequency of the spring-mass system be as far as possible from the disturbing frequency. Filtering of disturbing forces may be calculated as:

$$\% \text{ of force transmitted} = \frac{1}{(n_d/n)^2 - 1} \times 100$$

where  $n_d$  is the frequency of the disturbing force and  $n$  the natural frequency of the spring-mass system (Figure 5-13).

If  $n_d/n$  is less than 1, the denominator in Equation 5-14 should be changed to  $1 - (n_d/n)^2$ . Note that the frequency  $n$  in this equation is the frequency of the spring-mass system and not the natural spring frequency. In fact, the most commonly used equation neglects the spring weight and is only deflection dependent. The general equation is:

$$n = \frac{15.8}{\pi} \sqrt{\frac{gk}{P}} \text{ metric (or) } n = \frac{1}{2\pi} \sqrt{\frac{gk}{P}} \text{ English}$$

### 3.21 Special Springs

Previously in this section, design considerations for round wire helical compression springs of uniform diameter were discussed. These design techniques are modified below and applied to many special spring configurations. Special springs are chosen to fulfill a unique set of design criteria. Springs from rectangular wire and stranded wire as well as variable diameter springs with conical, hourglass, and barrel shapes are discussed below. Helpful guidelines for nested springs are also reviewed.

#### Rectangular Wire Springs

In applications where space is limited and particularly where solid height is restricted, springs designed from rectangular or keystone wire are often selected. Associated Spring manufactures hundreds of rectangular wire spring designs. These springs are commonly referred to as die springs and are available for immediate delivery.

Fig. 5-12. Velocity of an Object Propelled by a Compression Spring.

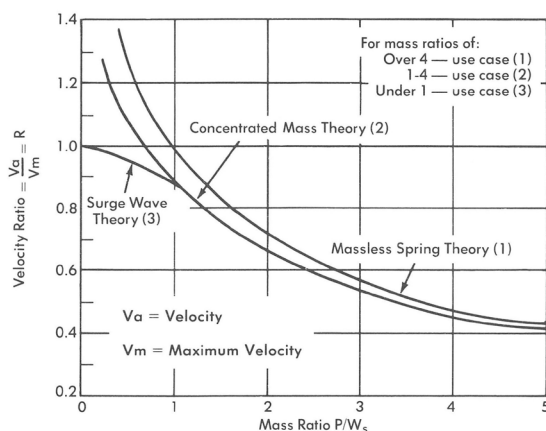
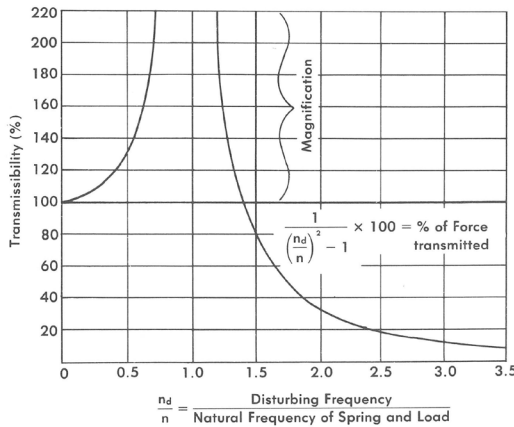


Fig. 5-13. Transmissibility of Spring Mounting.



Springs made from rectangular wire, with the width of the rectangle perpendicular to the spring axis, store more energy in a smaller space than equivalent round wire springs. Even though stress distribution around the rectangular cross-section is not as uniform as the round wire section, the ESC is higher because more material can be incorporated into the allocated space. Rectangular wire is more costly than round wire, but less costly than keystone wire. Keystone wire is processed specially so that deformation during spring winding or coiling causes the cross-section to become approximately rectangular. Distortion of the cross-section can be estimated from the equation presented in Figure 5-14. However, distortion depends upon the manufacturing technique employed, and this equation is at best an approximation. Axial dimensional change of the wire must always be considered when calculating solid heights of rectangular springs.

The rate for a compression spring made from rectangular wire is expressed as follows:

$$k = \frac{P}{f} = \frac{Gbt^3}{N_a D^3} K_2$$

Since the wire is loaded in torsion, the rate is the same whether the wire is wound on flat or on edge (Figure 5-15). Values for the constant  $K_2$  are shown in Figure 5-16. Stress is expressed as:

$$S = \frac{PD}{K_1 bt^2} K_E \text{ or } \frac{PD}{K_1 bt^2} K_F$$

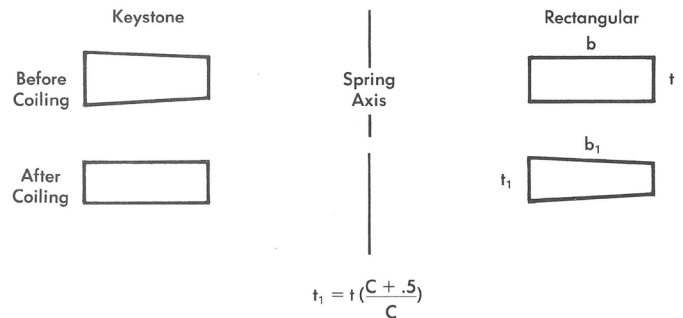
Values for  $K_1$  are shown in Figure 5-16, while values for the stress correction factor for springs wound on the flat ( $K_F$ ) and springs wound on edge ( $K_E$ ) are shown in Figure 5-17 and 5-18. When rectangular wire is produced by rolling round wire, or if the cross-section of the wire deviates significantly from a rectangle, additional correction factors are required. Whenever a round wire cannot be used because the solid height exceeds specifications, it is possible to try a rectangular wire coiled on edge where:

$$t = \frac{2d}{1 + b/t}$$

and  $d$  is equal to the wire diameter for the equivalent round wire spring. A typical value for a width-to-thickness ratio of 2 may be assumed in the initial design calculations.

### Stranded Wire Springs

Fig. 5-14. Wire Cross Section Before and After Coiling.



Long springs with many coils, when subjected to high rates of load application as in automatic weapons, encounter shock wave motion and can literally be torn apart. Stranded-wire springs are often the most successful solution to such problems because of the friction resistance between the strands.

To function properly, the helix of a spring must be opposite in direction to the helix of the strands,



so that the strands bind together when the spring is compressed. The stranded-wire spring may be wound with two, three, or more strands. Springs with four or more strands are made with a center wire core to assure necessary stability. Ends should be soldered, brazed, or welded to prevent unraveling.

Recognizing that a stranded-wire spring can be considered as single-wire springs arranged in parallel, spring rate is derived on the basis given by the following equation, where K = correction factor and n = number of strands:

$$k = \frac{KnGd^4}{8D^3N_a}$$

For a three-strand spring, K = 1.05.

An approximation for torsional stress in each

wire of the strand is given by Equation 5-4:

$$S = \frac{8PD}{\pi d^3} K_{W1}$$

Maximum allowable stress after set removal should not exceed 55% to 60% of the material tensile strength. Wire diameter ( $d_s$ ) for a single strand in a stranded wire spring is less than the wire diameter for a monolithic spring with the same mean diameter and rate.

Stress in the stranded-wire spring is also less

Number of Strands	Wire Size
2	$d_s$ greater than 0.79 d
3	$d_s$ greater than 0.69 d
4	$d_s$ greater than 0.63 d

than the stress in an equivalent monolithic spring.

Fig. 5-15. Rectangular Wire Compression Spring Wound on Flat or Edge.

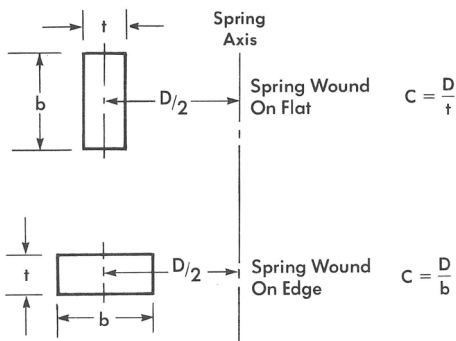


Fig. 5-16. Constants for Rectangular Wire in Torsion.

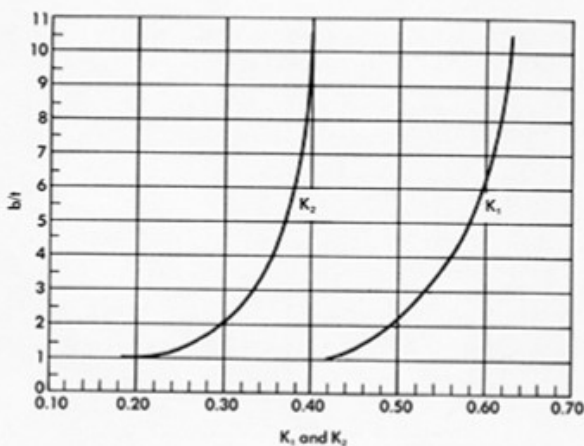


Fig. 5-17. Stress Correction Factors for Rectangular Wire Compression Springs Wound on Flat.

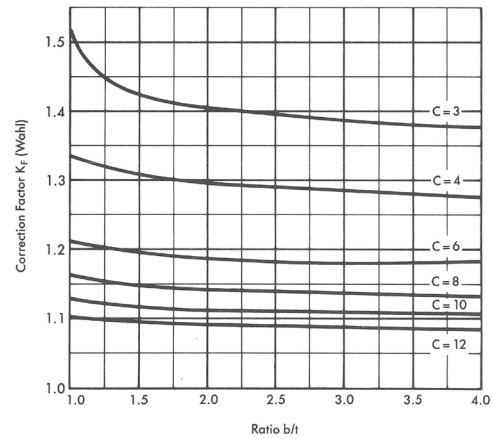
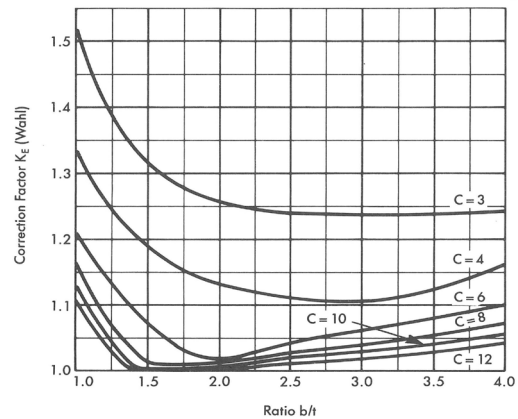


Fig. 5-18. Stress Correction Factors for Rectangular Wire Compression Springs Wound on Edge.



## Variable Diameter Springs

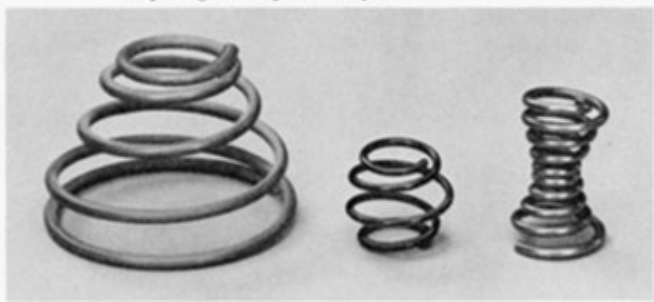
Conical, hourglass, and barrel-shaped springs (Figure 5-19) are used in applications requiring a low solid height, increased lateral stability, or resistance to surging. Conical springs can be designed so that each coil nests wholly or partly into an adjacent coil. Solid height can be as low as one wire diameter. Rate for conical springs usually increases with deflection (Figure 5-20) because the number of active coils decreases progressively as the spring approaches solid. By varying the pitch, conical springs can be designed to have a uniform rate. Rate for conical springs is calculated, as indicated previously, by considering the spring as many springs in series. Rate for each turn or fraction of a turn is calculated using Equation 5-2. Rate for a complete spring is then determined, remembering that the spring rate follows the series relationship given previously in Equation 5-3.

To calculate the highest stress at a given load, the mean diameter of the largest active coil at load is used. Solid height of a uniformly tapered, but not telescoping, spring with squared and ground ends made from round wire can be estimated from:

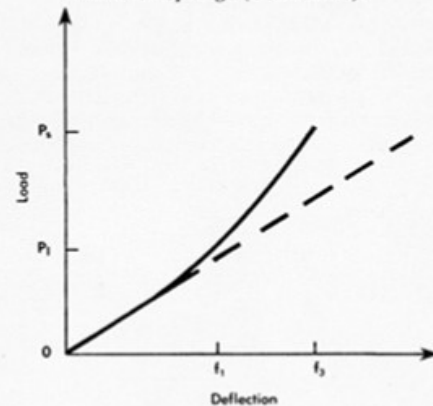
$$L_s = N_a \sqrt{d^2 - u^2} + 2d$$

where  $u$  = the O.D. large end minus the O.D. small end divided by  $2N_a$ .

**Fig. 5-19. Typical Conical, Barrel and Hourglass Springs Respectively.**



**Fig. 5-20. Typical Load Deflection Curve for Variable Diameter Springs (Solid Line).**



Designing a variable diameter spring so that adjacent coils rub against one another during deflection increases resistance to resonance phenomena but may also shorten spring life due to wear.

Barrel and hourglass springs are calculated as two conical springs in series.

## Variable Pitch Springs

Variable pitch springs (Figure 5-21) are used to achieve a variable rate similar to that shown in Figure 520 or in dynamic applications where the cyclic rate of load application is near the natural spring frequency. As turns of lesser pitch become inactive during deflection, the natural frequency of a spring changes. Throughout the cycle, the spring has a spectrum of frequency response and not a single resonant frequency. Thus, surging and spring resonance are minimized.

## Nested Compression Springs

Helical compression springs are often used in combination because of space limitations and resonance considerations. A nest of compression springs can store more energy but will have lower natural frequencies than a single equivalent spring. Nested springs are not recommended when the diametral space is so restricted that a single spring would have an index of 5 or less. The following design practices

apply to nested springs:

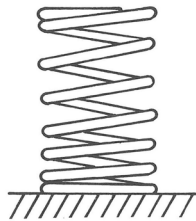
- To prevent internesting, the springs should be wound alternately left- and right-handed.
- Clearance between springs must be at least twice the diameter tolerance.
- The most efficient distribution of load between individual springs varies with their indexes and the clearances between them. For a first approximation in designing a nest with two springs, one-third of the load should be on the inner spring and two-thirds on the outer spring.
- Solid heights and free heights should be about the same for all springs.

These practices result in springs with approximately the same index.

### 3.22 Commercial Tolerances

Standard commercial tolerances for free length, diameter, and load are presented in Tables 5-4, 5-5, and 5-6. Tolerance on squareness is  $3^\circ$ . These tolerances represent a good trade-off between manufacturing costs and performance in most applications. Certain premium spring materials and processing methods can be used to achieve tighter tolerances. If the application requires tighter tolerances, the required tolerance levels should be discussed with an Associated Spring engineer.

*Fig. 5-21. Typical Variable Pitch Helical Compression Spring.*



For fatigue applications, spring life is often specified. Unless otherwise stated, life is interpreted as the  $S_{10}$  life. This is the life at which 90% of the springs are expected to survive with a 50% confidence level based on Weibull analysis.

### 3.23 Acceptable Quality Level (AQL)

Quality levels are often expressed by an AQL (for example MIL-STD-105, Sampling Procedures and Tables for Inspection by Attributes). Only critical attributes should be subject to an AQL on the drawing. Unnecessarily tight AQLs will increase manufacturing and inspection costs. If tolerances must be close for proper functioning and if, for instance, nonconforming parts can be discarded at assembly, a standard AQL will minimize the parts cost. Springs assembled automatically often require tight AQL on dimensions, while springs used in instruments and critical machines often require tight AQL on loads and life. A close liaison between Associated Spring engineers and the designer during design and prototype phases is the best way to ensure optimum quality.

### 3.24 Packaging

Normally, compression springs and other custom parts are packaged in bulk. Compression springs with high pitch angles and large indexes are subject to tangling. Tangling not only makes it difficult to separate springs upon arrival but can also cause distortion. Special packaging systems—such as the Spring Flow™ system where springs are packaged in rows (Figure 5-22)—can prevent tangling. Another method is to place springs on adhesive-coated corrugated panels. There are many other packaging methods used to prevent tangling and reduce shipment bulk.

**Table 5-4. Free Length Tolerances of Squared and Ground Helical Compression Springs.**

Number of Active Coils per mm(in.)	Tolerances: ±mm/mm (in./in.) of Free Length						
	Spring Index (D/d)						
	4	6	8	10	12	14	16
0.02 (0.5)	0.010	0.011	0.012	0.013	0.015	0.016	0.016
0.04 (1)	0.011	0.013	0.015	0.016	0.017	0.018	0.019
0.08 (2)	0.013	0.015	0.017	0.019	0.020	0.022	0.023
0.2 (4)	0.016	0.018	0.021	0.023	0.024	0.026	0.027
0.3 (8)	0.019	0.022	0.024	0.026	0.028	0.030	0.032
0.5 (12)	0.021	0.024	0.027	0.030	0.032	0.034	0.036
0.6 (16)	0.022	0.026	0.029	0.032	0.034	0.036	0.038
0.8 (20)	0.023	0.027	0.031	0.034	0.036	0.038	0.040

For springs less than 12.7 mm (0.500") long, use the tolerances for 12.7 mm (0.500"). For closed ends not ground, multiply above values by 1.7.

### 3.25 How to Specify

There are many ways to specify compression springs. Because the number of variables is large, it is useful for the designer to use a specification checklist.

### 3.26 Compression Spring Design Example

Given: Squared and ground compression spring to work in a hole  $D_H = 40$  mm (1.575") and exert  $P_1 = 275$  N (61.8 lbf) at a height of  $L_1 = 60$  mm (2.362") and  $P_2 = 500$  N (112 lbf) at a height of  $L_2 = 50$  mm (1.969"). Application: static at room temperature. Material: oil-tempered wire ASTM A229. Spring must not set when compressed to solid height.

A. First estimate the wire diameter by solving equation (Equation 5-4) using approximate values for unknown factors and  $K_{W1} = 1$ .

Then, calculate O.D. and D.

Substitute this wire size in the load deflection equation (Equation 5-2) and solve for  $N_a$ .

Repeat this process until a satisfactory solution is obtained.

1. Rearranging Equation 5-4 for uncorrected stress:

$$d = \sqrt[3]{\frac{2.55 P_2 D}{S_2}}$$

2. Assume tensile strength of ASTM A229 is 1500 MPa and  $S_2 = 700$  MPa uncorrected:

$$d = \sqrt[3]{\frac{2.55 \times 500 \times 40}{700}} = 4.2 \text{ mm}$$

3. For clearance, assume O.D. = 0.95  $D_H$ :

$$\text{O.D.} = 0.95 \times 40 = 38.0 \text{ mm}$$

$$D = 38.0 - 4.2 = 33.8 \text{ mm}$$

$$C = \frac{D}{d} = \frac{33.8}{4.2} = 8.0$$

$$\text{Rate} = k = \frac{500 - 275}{60 - 50} = 22.5 \text{ N/mm}$$

$$N_a = \frac{Gd^4}{8D^3k}$$

$$N_a = \frac{(7.93 \times 10^4)(4.2)^4}{8(33.8)^3(22.5)} = 3.55$$

**Table 5-5. Coil Diameter Tolerances of Helical Compression and Extension Springs.**

Wire Dia., mm(in.)	Tolerances: ±mm (in.)						
	Spring Index (D/d)						
	4	6	8	10	12	14	16
0.38 (0.015)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.13 (0.005)	0.15 (0.006)	0.18 (0.007)
0.58 (0.023)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.20 (0.008)	0.25 (0.010)
0.89 (0.035)	0.05 (0.002)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.23 (0.009)	0.28 (0.011)	0.33 (0.013)
1.30 (0.051)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.25 (0.010)	0.30 (0.012)	0.38 (0.015)	0.43 (0.017)
1.93 (0.076)	0.10 (0.004)	0.18 (0.007)	0.25 (0.010)	0.33 (0.013)	0.41 (0.016)	0.48 (0.019)	0.53 (0.021)
2.90 (0.114)	0.15 (0.006)	0.23 (0.009)	0.33 (0.013)	0.46 (0.018)	0.53 (0.021)	0.64 (0.025)	0.74 (0.029)
4.34 (0.171)	0.20 (0.008)	0.30 (0.012)	0.43 (0.017)	0.58 (0.023)	0.71 (0.028)	0.84 (0.033)	0.97 (0.038)
6.35 (0.250)	0.28 (0.011)	0.38 (0.015)	0.53 (0.021)	0.71 (0.028)	0.90 (0.035)	1.07 (0.042)	1.24 (0.049)
9.53 (0.375)	0.41 (0.016)	0.51 (0.020)	0.66 (0.026)	0.94 (0.037)	1.17 (0.046)	1.37 (0.054)	1.63 (0.064)
12.70 (0.500)	0.53 (0.021)	0.76 (0.030)	1.02 (0.040)	1.57 (0.062)	2.03 (0.080)	2.54 (0.100)	3.18 (0.125)



B. Find amount of space left between  $L_2$  and  $L_s$ :

1. Compare to  $f_2$ .
2. Find the corrected stress at solid height.
3. Compare to tensile strength of material. See Figure 3-3, page 19.

$$L_s = 5.55 \times 4.2 = 23.3$$

$$L_f = \frac{P_1}{k} + L_1 = \frac{275}{22.5} + 60 = 72.2 \text{ mm}$$

$$f_2 = 72.2 - 50 = 22.2 \text{ mm}$$

$$L_2 - L_s = 50 - 23.3 = 26.7 \text{ mm}$$

$$f_s = 72.2 - 23.3 = 48.9 \text{ mm}$$

$$15\% \text{ of } 48.9 = 7.3 \text{ mm}$$

$$K_{w1} = \frac{4C - 1}{4C - 4} + \frac{.615}{C} = 1.18$$

$$P_s = f_s \times k = 48.9 \times 22.5 = 1100 \text{ N}$$

$$S_s = \frac{2.55P_s D}{d^3} K_{w1} = \frac{(2.55)(1100)(33.8)(1.18)}{(4.2)^3} \\ = 1510 \text{ MPa}$$

4. Tensile strength of 4.2 mm diameter wire = 1400 MPa. Before set is removed, maximum allowable torsional stress is 50% of TS or 700 MPa (Table 5-2, page 35).  $S_s = 1510$  is greater than 700 MPa, and the spring will set.

C. Because  $(L_2 - L_s) = 26.7 > 0.15f_s = 7.3$ , there is more space available. Try a larger preferred wire size (Table 3-8, page 20) of 4.8 mm.

TS = 1400 MPa,  $D = 38.0 - 4.8 = 33.2$  mm,  $C = 6.9$

$$N_a = \frac{(7.93 \times 10^4)(4.8)^4}{8(33.2)^3(22.5)} = 6.4$$

$$L_s = 8.4 \times 4.8 = 40.3 \text{ mm}$$

$$L_2 - L_s = 50 - 40.3 = 9.7 \text{ mm}$$

$$f_s = 72.2 - 40.3 = 31.9 \text{ mm}$$

$$(L_2 - L_s) = 9.7 > 0.15 f_s = 4.8 \text{ mm}$$

$$P_s = (31.9)(22.5) = 718 \text{ N}$$

$$K_{w1} = \frac{(4)(6.9) - 1}{(4)(6.9) - 4} + \frac{.615}{6.9} = 1.22$$

$$S_s = \frac{(2.55)(718)(33.2)(1.22)}{(4.8)^3} = 671 \text{ MPa}$$

$$S_s = 671 \text{ MPa or } \frac{671}{1400} \times 100 = 48\% \text{ of TS}$$

**COMPRESSION SPRING SPECIFICATION CHECKLIST**

(Fill in required data only)

Material: \_\_\_\_\_

**Working Conditions:**

To work in \_\_\_\_\_mm(in.) diameter hole

To work over \_\_\_\_\_mm(in. diameter shaft

Load \_\_\_\_\_N (lbf), ± \_\_\_\_\_N (lbf)

at \_\_\_\_\_mm(in.)

Load \_\_\_\_\_N (lbf), ± \_\_\_\_\_N (lbf)

at \_\_\_\_\_mm(in.)

Rate \_\_\_\_\_N/mm (lbf/in.), ± \_\_\_\_\_N/mm (lbf/in.)

between \_\_\_\_\_mm(in.) and \_\_\_\_\_mm(in.)

Maximum solid height \_\_\_\_\_mm(in.)

Direction of coil (right-hand, left-hand or optional) \_\_\_\_\_

Type of ends \_\_\_\_\_

Allowable relaxation \_\_\_\_\_% Hours/days \_\_\_\_\_

Impact loading \_\_\_\_\_mm/sec (in./sec)

Frequency of loading \_\_\_\_\_Hz

Required life \_\_\_\_\_cycles.

Required reliability (see Section 4) \_\_\_\_\_

**Special Information:**

Squareness \_\_\_\_\_ Parallelism \_\_\_\_\_

Finish \_\_\_\_\_

Maximum operating temperature \_\_\_\_\_°C(°F)

Operating environment \_\_\_\_\_

Electrical/magnetic \_\_\_\_\_

**Design Data (Reference):**

Wire diameter \_\_\_\_\_mm(in.)

Outside diameter \_\_\_\_\_mm(in.)

Inside diameter \_\_\_\_\_mm(in.)

Free length \_\_\_\_\_mm(in.)

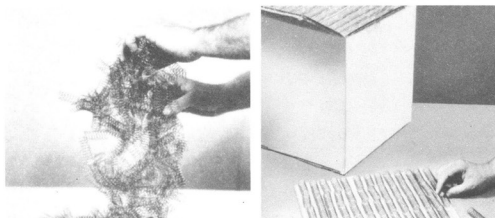
Total number of coils \_\_\_\_\_

**Table 5-6. Load Tolerances of Helical Compression Springs.**

Length Tolerance ±mm (in.)	Tolerances: ±% of Load. Start with Tolerance from Table 5-4, page 42 Multiplied by L <sub>F</sub> .														
	Deflection from Free Length to Load, mm (in.)														
	1.27 (0.050)	2.54 (0.100)	3.81 (0.150)	5.08 (0.200)	6.35 (0.250)	7.62 (0.300)	10.2 (0.400)	12.7 (0.500)	19.1 (0.750)	25.4 (1.00)	38.1 (1.50)	50.8 (2.00)	76.2 (3.00)	102 (4.00)	152 (6.00)
0.13 (0.005)	12.	7.	6.	5.	—	—	—	—	—	—	—	—	—	—	—
0.25 (0.010)	—	12.	8.5	7.	6.5	5.5	5.	—	—	—	—	—	—	—	—
0.51 (0.020)	—	22.	15.5	12.	10.	8.5	7.	6.	5.	—	—	—	—	—	—
0.76 (0.030)	—	—	22.	17.	14.	12.	9.5	8.	6.	5.	—	—	—	—	—
1.0 (0.040)	—	—	—	22.	18.	15.5	12.	10.	7.5	6.	5.	—	—	—	—
1.3 (0.050)	—	—	—	—	22.	19.	14.5	12.	9.	7.	5.5	—	—	—	—
1.5 (0.060)	—	—	—	—	25.	22.	17.	14.	10.	8.	6.	5.	—	—	—
1.8 (0.070)	—	—	—	—	—	25.	19.5	16.	11.	9.	6.5	5.5	—	—	—
2.0 (0.080)	—	—	—	—	—	—	22.	18.	12.5	10.	7.5	6.	5.	—	—
2.3 (0.090)	—	—	—	—	—	—	25.	20.	14.	11.	8.	6.	5.	—	—
2.5 (0.100)	—	—	—	—	—	—	—	22.	15.5	12.	8.5	7.	5.5	—	—
5.1 (0.200)	—	—	—	—	—	—	—	—	—	22.	15.5	12.	8.5	7.	5.5
7.6 (0.300)	—	—	—	—	—	—	—	—	—	—	22.	17.	12.	9.5	7.
10.2 (0.400)	—	—	—	—	—	—	—	—	—	—	—	21.	15.	12.	8.5
12.7 (0.500)	—	—	—	—	—	—	—	—	—	—	—	25.	18.5	14.5	10.5

First load test at not less than 15% of available deflection.  
Final load test at not more than 85% of available deflection.

**Fig. 5-22. Tangled Helical Compression Springs (Left) and Spring Flow Packaging.**



**Final Design Specifications:**

Material: ASTM A229  
Wire Diameter d: 4.8 mm (0.189") Reference  
O.D.: 38.0 ± 0.4 mm (1.500 ± 0.050")  
Free Length L<sub>F</sub>: 72.2 mm (2.843") Reference  
Test Height L<sub>1</sub>: 60 mm (2.362")  
Test Height L<sub>2</sub>: 50 mm (1.969")  
P<sub>1</sub> Load at L<sub>1</sub>: 275 N (61.8 lbf) ± 11.0%  
P<sub>2</sub> Load at L<sub>2</sub>: 500 N (112 lbf) ± 7%  
Final Design Stress S<sub>s</sub>: 671 MPa (97,300 psi) or 48% TS  
N<sub>i</sub>: 8.4



## 4 HELICAL EXTENSION SPRINGS

### 4.1 Introduction

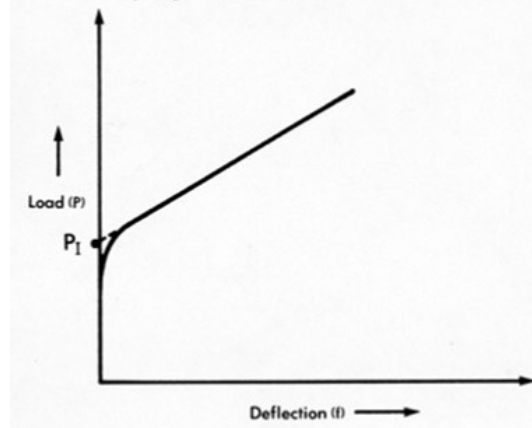
Helical extension springs store energy and exert a pulling force. Usually, they are made from round wire and are close-wound with initial tension. Typical applications include tape cassette players, balance scales, toys, garage doors, automatic washing machines, and various types of spring tensioning devices.

Helical extension springs are stressed in torsion in the body. Design procedures for the body are similar to those discussed previously for compression springs (Section 5) with the following major exceptions. Most helical extension springs are coiled with initial tension, equal to the minimum force required to separate adjacent coils. Helical extension springs do not normally have set removed. Furthermore, unlike compression springs, extension springs do not have a solid stop to prevent overloading. For these reasons, design stress levels are generally lower for extension than for compression springs. A special type of extension spring, known as a drawbar spring (Figure 7-1), has a solid stop. It is essentially a compression spring with special hooks.

**Fig. 7-1. Drawbar Spring Provides a Solid Stop.**



**Fig. 7-2. Load Deflection Curve for a Helical Extension Spring with Initial Tension.**



The pulling force exerted by an extension spring body is transmitted to mating parts through hooks or loops. When stresses in the hooks are higher than in the spring body, the hooks limit spring performance.

Associated Spring includes hundreds of different extension spring designs with full twist loops in its SPEC line of stock springs. These extension springs are made from either music wire or stainless steel and are pre-engineered to meet a wide range of applications.

### 4.2 Initial Tension

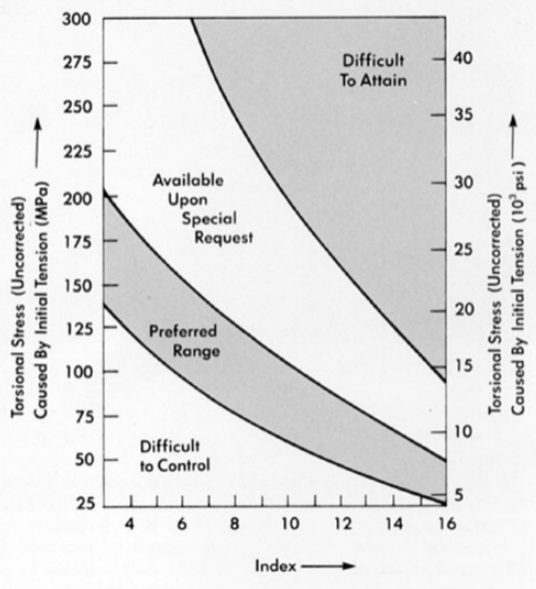
Initial tension in an extension spring is measured according to the procedure illustrated in Figure 7-2. The linear portion of the load deflection curve is extrapolated to zero deflection. The point of intersection on the ordinate is initial tension  $P_1$ . The amount of initial tension that can be put into a spring depends upon its index, material, method of manufacture, and post-coiling stress-relief treatment. Occasionally, in critical applications when stress is high, a high stress-relief temperature is required to minimize unfavorable residual stresses due to coiling or

forming the hooks. High temperature stress relief reduces the amount of initial tension. Typical values of initial tension are shown in Figure 7-3. High strength materials, such as small diameter music wire, are able to support higher levels of initial tension than low strength materials such as large diameter hard-drawn wire.

### 4.3 Types of Ends

Extension springs require a method of attachment to other parts in an assembly. A wide variety of ends have been developed and used successfully for many years—for example, threaded inserts, swivel hooks, twist loops, side loops, cross-center loops, and extended hooks. Loops are attachment ends that have small gaps (Figure 7-6), while hooks are loops with a large gap. In fact, the variety of ends is almost unlimited. The most common configurations are those that can be formed during the springmaking operation.

Fig. 7-3. Torsional Stress Due to Initial Tension as a Function of Index in Helical Extension Springs.



Typical types include twist, cross center, side loops, and extended hooks (Figure 7-4). Many of

these configurations are made by bending the last coils of an extension spring to form loops. Most special hooks are formed from straight sections of wire on the so-called “tangent ends” of an extension spring body.

Guidelines for the lengths of common loops are presented in Figure 7-4. Although other configurations and lengths are available, common loops of preferred lengths are generally the most economical. If possible, a spring should be designed with one or both loops at the preferred length. For example, if a design requires a total loop length of equal to five times the I.D., a popular choice is one twist loop with a length equal to the I.D. and one extended loop with a length equal to four times the I.D. Whenever possible for extended loops, the designer should allow for a straight section approximately three wire diameters long at the end of the wire (A, Figure 7-4). Loops at each end can be made with a controlled angular relationship. Specifying an angular relationship may add to the cost; therefore, whenever an application permits, a random angular relationship should be allowed. Production of special end configuration may involve tool charges and generally results in increased costs.

Fig. 7-4. Common End Configurations for Helical Extension Springs.

Type	Configurations	Recommended Length* Min.- Max.
Twist Loop or Hook		0.5-1.7 I.D.
Cross Center Loop or Hook		1 I.D.
Side Loop or Hook		0.9-1.0 I.D.
Extended Hook		1.1 I.D. and up, as required by design
Special Ends		As required by design

\* Length is distance from last body coil to inside of end. I.D. is inside diameter of adjacent coil in spring body.

Stresses in loops are often higher than in the spring body. This limits spring performance, particularly in cyclic applications. Generous bend radii in loops and reduced end-coil diameters



are two methods frequently employed to reduce stresses. In a full twist loop, stress reaches a maximum at point A in bending and a maximum in torsion at point B (Figure 7-5). Stress at these locations is complex, but can be estimated with reasonable accuracy by:

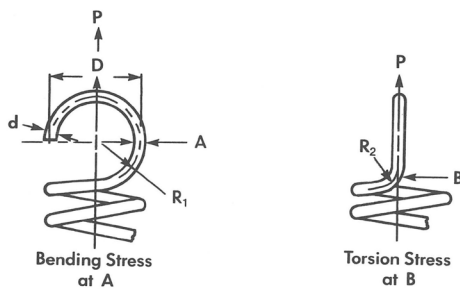
$$(7-1) \quad S_A = \frac{16DP}{\pi d^3} K_1 + \frac{4P}{\pi d^2} \text{ bending}$$

$$(7-2) \quad \text{where } K_1 = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \text{ and } C_1 = \frac{2R_1}{d}$$

$$S_B = \frac{8DP}{\pi d^3} \left( \frac{4C_2 - 1}{4C_2 - 4} \right) \text{ and } C_2 = \frac{2R_2}{d} \text{ torsion}$$

Recommended practice is to make  $C_2$  greater than four.

Fig. 7-5. Location of Maximum Bending and Torsion Stresses in Twist Loops.



#### 4.4 Extension Spring Dimensions

Free length of an extension spring is the distance between the inner surfaces of the ends (Figure 7-6). It is equal to the spring body length plus ends, where spring body length is given by  $L_{\text{body}} = d(N + 1)$ . The gap, which is sometimes referred to as hook or loop opening, can be varied by the springmaker. Certain manufacturing processes require a minimum gap, and the designer should consult Associated Spring engineers if a gap must be less than one-half wire diameter. The number of active coils in a spring is approximately equal to the number of coils in its body. For springs with threaded inserts or swivel hooks, the number of active coils is less than the total coils in the body. Hooks and loops add to

the number of active coils. Allowances of  $0.1 N_a$  are occasionally made for one-half twist loops. Allowances as large as  $0.5 N_a$  can be made for some cross center, full twist, and extended loops.

#### 4.5 Design Equations

Design equations for extension springs are similar to compression springs. The rate is given by:

$$(7-3) \quad k = \frac{P - P_1}{f} = \frac{Gd^4}{8D^3N_a}$$

Where  $P_1$  is initial tension. Stress is given by:

$$(7-4) \quad S = \frac{8PD}{\pi d^3} K_w$$

Dynamic considerations discussed previously in Section 5 are generally applicable to extension springs. Natural frequency when one end is fixed is given by:

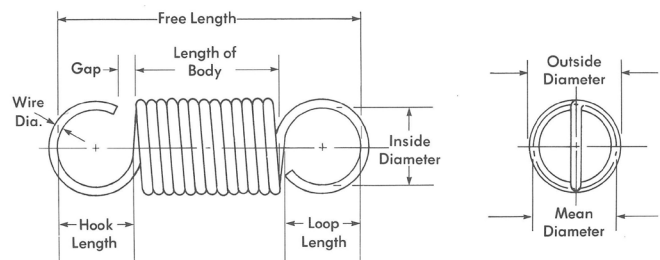
$$(7-5) \quad n = \frac{5.6 \times 10^2 d}{D^2 N_a} \sqrt{\frac{Gg}{\rho}} \text{ metric}$$

$$\text{for steel } n = \frac{1.76 \times 10^5 d}{N_a D^2} \text{ metric}$$

$$n = \frac{d}{18D^2 N_a} \sqrt{\frac{Gg}{\rho}} \text{ English}$$

$$\text{for steel } n = \frac{7000d}{N_a D^2} \text{ English}$$

Fig. 7-6. Typical Extension Spring Dimensions.



## 4.6 Choice of Operating Stress—Static

Recommended maximum stresses for extension springs used in static applications (Table 7-1) are similar to levels recommended for compression springs without set removal. For springs that cannot be adequately stress-relieved due to high initial tension requirements, the maximum recommended stress in the body should be reduced to that recommended for their ends. Maximum recommended stress in the ends is lower than in the body because the wire is often stretched, marked, or distorted during loop-making.

## 4.7 Choice of Operating Stress—Cyclic

Maximum recommended stresses for extension springs used in cyclic applications are presented in Table 7-2. These data are for stress-relieved springs with low levels of initial tension.

**Table 7-2.** Maximum Allowable Stresses for ASTM A228 and Type 302 Stainless Steel Helical Extension Springs in Cyclic Applications.

Number of Cycles	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
10 <sup>5</sup>	36	34	51
10 <sup>6</sup>	33	30	47
10 <sup>7</sup>	30	28	45

This information is based on the following conditions: not shot-peened, no surging and ambient environment with a low temperature heat treatment applied. Stress ratio = 0.

## 4.8 Clearances

Extension springs, when deflected, do not require central arbors or holes to prevent buckling. When a spring is dynamically loaded or unloaded suddenly (as a cam drop-off), it may vibrate laterally, inducing additional stresses. If clearance is not allowed, this lateral vibration may be noisy and result in premature failure from abrasion of the spring or adjacent parts.

## 4.9 Tolerances

Since requesting close tolerances can increase manufacturing costs, only those characteristics that are critical to spring performance should have tolerances specified. Commercial free length, angular relationship of ends, and load tolerances are presented in Tables 7-3, 7-4, and 7-5 respectively. Outside diameter tolerances are the same as for compression springs (Table 5-5). These tables should be used only as a guide, since some manufacturing operations have different process capabilities that can cause variations in tolerance values. For special applications requiring closer tolerances, consult Associated Spring engineers.

**Table 7-1.** Maximum Allowable Stresses ( $K_{wt}$  Corrected) for Helical Extension Springs in Static Applications.

Materials	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
Patented, cold-drawn or hardened and tempered carbon and low alloy steels	45-50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55

This information is based on the following conditions: set not removed and low temperature heat treatment applied. For springs that require high initial tension, use the same percent of tensile strength as for end.

**Table 7-3. Commercial Free Length Tolerances for Helical Extension Springs With Initial Tension.**

Spring Free Length (inside hooks) mm (in.)	Tolerance ± mm (in.)
Up to 12.7 (0.500)	0.51 (0.020)
Over 12.7 to 25.4 (0.500 to 1.00)	0.76 (0.030)
Over 25.4 to 50.8 (1.00 to 2.00)	1.0 (0.040)
Over 50.8 to 102 (2.00 to 4.00)	1.5 (0.060)
Over 102 to 203 (4.00 to 8.00)	2.4 (0.093)
Over 203 to 406 (8.00 to 16.0)	4.0 (0.156)
Over 406 to 610 (16.0 to 24.0)	5.5 (0.218)

### 4.10 How to Specify

For minimum cost, it is important to specify springs properly. The helical extension springs specification checklist is presented as a guide.

**Table 7-4. Tolerances on Angular Relationship of Extension Spring Ends.**

Angular Tolerance per Coil: ± Degrees									
Index									
4	5	6	7	8	9	10	12	14	16
0.75	0.9	1.1	1.3	1.5	1.7	1.9	2.3	2.6	3

For example, tolerance for a 10-coil spring with an index of 8 is  $10 \times \pm 1.5 = \pm 15^\circ$ .

If angular tolerance is greater than  $\pm 45^\circ$ , or if closer tolerances than indicated must be held, consult with Associated Spring.

#### HELICAL EXTENSION SPRINGS SPECIFICATION CHECKLIST

(Fill in required data only.)

Material \_\_\_\_\_

Working Conditions:

Maximum outside diameter \_\_\_\_\_mm(in.)

Initial tension \_\_\_\_\_N(lbf)

Load \_\_\_\_\_N(lbf), ± \_\_\_\_\_  
at \_\_\_\_\_length mm(in.)

Load \_\_\_\_\_N(lbf), ± \_\_\_\_\_  
at \_\_\_\_\_length mm(in.)

Impact Loading \_\_\_\_\_m/sec(in./sec)

Rate \_\_\_\_\_N/mm(lbf/in)

Maximum extended length in service \_\_\_\_\_mm(in.)  
\_\_\_\_\_mm(in.) during installation

Direction of coil: right \_\_\_\_\_ left \_\_\_\_\_  
optional \_\_\_\_\_

Type of ends \_\_\_\_\_

Position of ends and tolerance \_\_\_\_\_

Gap opening and tolerance \_\_\_\_\_mm(in.)

Suggested Design Data:

Wire diameter \_\_\_\_\_mm(in.)

Outside diameter \_\_\_\_\_mm(in.)

Total number of coils \_\_\_\_\_

Free length inside ends \_\_\_\_\_mm(in.)

Special Information:

Finish \_\_\_\_\_

Maximum operating temperature \_\_\_\_\_°C(°F)

Operating environment \_\_\_\_\_

Frequency of Loading \_\_\_\_\_hertz

Required life \_\_\_\_\_cycles

Required Reliability (see Section 4) \_\_\_\_\_



Table 7-5. Load Tolerances for Helical Extension Springs.

Index	Body Length Divided by Deflection	Tolerances: +/- Percent of Specified Load (Closer tolerances may require additional operations.)										
		Wire Diameter: mm(in.)										
$\frac{D}{d}$	Lb f	0.38 (0.015)	0.56 (0.022)	0.81 (0.032)	1.0 (0.041)	1.6 (0.062)	2.3 (0.092)	3.2 (0.125)	4.7 (0.187)	6.4 (0.250)	4.5 (0.375)	11.1 (0.437)
4	12	20.0	18.5	17.6	16.9	16.2	15.5	15.0	14.3	13.8	13.0	12.6
	8	18.5	17.5	16.7	15.8	15.0	14.5	14.0	13.2	12.5	11.5	11.0
	6	16.8	16.1	15.5	14.7	13.8	13.2	12.7	11.8	11.2	9.9	9.4
	4.5	15.0	14.7	14.1	13.5	12.6	12.0	11.5	10.3	9.7	8.4	7.9
	2.5	13.1	12.4	12.1	11.8	10.6	10.0	9.1	8.5	8.0	6.8	6.2
	1.5	10.2	9.9	9.3	8.9	8.0	7.5	7.0	6.5	6.1	5.3	4.8
	0.5	6.2	5.4	4.8	4.6	4.3	4.1	4.0	3.8	3.6	3.3	3.2
5	12	17.8	16.5	15.7	15.5	14.8	14.1	13.5	12.8	12.3	12.0	11.5
	8	16.8	15.7	14.9	14.3	13.5	13.0	12.5	11.7	11.2	10.6	10.1
	6	15.8	14.8	13.8	13.2	12.3	11.8	11.4	10.7	10.0	9.3	8.7
	4.5	14.2	13.1	12.3	11.7	11.1	10.6	10.2	9.6	8.8	8.0	7.4
	2.5	12.3	11.3	10.8	10.0	9.6	9.0	8.7	8.1	7.5	6.5	6.1
	1.5	10.0	9.3	8.9	8.4	8.0	7.7	7.2	6.5	6.2	5.0	4.5
	0.5	6.2	5.4	4.9	4.6	4.4	4.2	4.1	3.9	3.7	3.4	3.3
6	12	17.0	15.5	14.6	14.1	13.5	13.1	12.7	12.0	11.5	11.2	10.7
	8	16.2	14.7	13.9	13.4	12.6	12.2	11.7	11.0	10.5	10.0	9.5
	6	15.2	14.0	12.9	12.3	11.6	10.9	10.7	10.0	9.4	8.8	8.3
	4.5	13.7	12.4	11.5	11.0	10.5	10.0	9.6	9.0	8.3	7.6	7.1
	2.5	11.9	10.8	10.2	9.8	9.4	9.0	8.5	7.9	7.2	6.2	6.0
	1.5	9.9	9.0	8.3	7.7	7.3	7.0	6.7	6.4	6.0	4.9	4.7
	0.5	6.3	5.5	4.9	4.7	4.5	4.3	4.1	4.0	3.7	3.5	3.4
8	12	15.8	14.3	13.1	13.0	12.1	12.0	11.5	10.8	10.2	10.0	9.5
	8	15.0	13.7	12.5	12.1	11.4	11.0	10.6	10.1	9.4	9.0	8.6
	6	14.2	13.0	11.7	11.2	10.6	10.0	9.7	9.3	8.6	8.1	7.6
	4.5	12.8	11.7	10.7	10.1	9.7	9.0	8.7	8.3	7.8	7.2	6.6
	2.5	11.2	10.2	9.5	8.8	8.3	7.9	7.7	7.4	6.9	6.1	5.6
	1.5	9.5	8.6	7.8	7.1	6.9	6.7	6.5	6.2	5.8	4.9	4.5
	0.5	6.3	5.6	5.0	4.8	4.5	4.4	4.2	4.1	3.9	3.6	3.5
10	12	14.8	13.3	12.0	11.9	11.1	10.9	10.5	9.9	9.3	9.2	8.8
	8	14.2	12.8	11.6	11.2	10.5	10.2	9.7	9.2	8.6	8.3	8.0
	6	13.4	12.1	10.8	10.5	9.8	9.3	8.9	8.6	8.0	7.6	7.2
	4.5	12.3	10.8	10.0	9.5	9.0	8.5	8.1	7.8	7.3	6.8	6.4
	2.5	10.8	9.6	9.0	8.4	8.0	7.7	7.3	7.0	6.5	5.9	5.5
	1.5	9.2	8.3	7.5	6.9	6.7	6.5	6.3	6.0	5.6	5.0	4.6
	0.5	6.4	5.7	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.8	3.7
12	12	14.0	12.3	11.1	10.8	10.1	9.8	9.5	9.0	8.5	8.2	7.9
	8	13.2	11.8	10.7	10.2	9.6	9.3	8.9	8.4	7.9	7.5	7.2
	6	12.6	11.2	10.2	9.7	9.0	8.5	8.2	7.9	7.4	6.9	6.4
	4.5	11.7	10.2	9.4	9.0	8.4	8.0	7.6	7.2	6.8	6.3	5.8
	2.5	10.5	9.2	8.5	8.0	7.8	7.4	7.0	6.6	6.1	5.6	5.2
	1.5	8.9	8.0	7.2	6.8	6.5	6.3	6.1	5.7	5.4	4.8	4.5
	0.5	6.5	5.8	5.3	5.1	4.9	4.7	4.5	4.3	4.2	4.0	3.3
14	12	13.1	11.3	10.2	9.7	9.1	8.8	8.4	8.1	7.6	7.2	7.0
	8	12.4	10.9	9.8	9.2	8.7	8.3	8.0	7.6	7.2	6.8	6.4
	6	11.8	10.4	9.3	8.8	8.3	7.7	7.5	7.2	6.8	6.3	5.9
	4.5	11.1	9.7	8.7	8.2	7.8	7.2	7.0	6.7	6.3	5.8	5.4
	2.5	10.1	8.8	8.1	7.6	7.1	6.7	6.5	6.2	5.7	5.2	5.0
	1.5	8.6	7.7	7.0	6.7	6.3	6.0	5.8	5.5	5.2	4.7	4.5
	0.5	6.6	5.9	5.4	5.2	5.0	4.8	4.6	4.4	4.3	4.2	4.0
16	12	12.3	10.3	9.2	8.6	8.1	7.7	7.4	7.2	6.8	6.3	6.1
	8	11.7	10.0	8.9	8.3	7.8	7.4	7.2	6.8	6.5	6.0	5.7
	6	11.0	9.6	8.5	8.0	7.5	7.1	6.9	6.5	6.2	5.7	5.4
	4.5	10.5	9.1	8.1	7.5	7.2	6.8	6.5	6.2	5.8	5.3	5.1
	2.5	9.7	8.4	7.6	7.0	6.7	6.3	6.1	5.7	5.4	4.9	4.7
	1.5	8.3	7.4	6.6	6.2	6.0	5.8	5.6	5.3	5.1	4.6	4.4
	0.5	6.7	5.9	5.5	5.3	5.1	5.0	4.8	4.6	4.5	4.3	4.1



#### 4.11 Extension Spring Design Example

A spring is to be incorporated into an overload circuit breaker. It is to be pre-loaded at length  $L_1 = 25.00$  mm (0.984") and must exert a load of 17.5N (3.93 lbf),  $\pm 15\%$ , when the circuit breaker is closed. If overload occurs, the circuit breaker is tripped, and the spring is extended to a length  $L_2 = 29.00$  mm (1.142"). The load must be 30 N  $\pm 12\%$  to operate a lock, preventing accidental resetting before the malfunction is corrected. Either twist or extended loops with generous radii are satisfactory. Because of surrounding components, the maximum O.D. is 7 mm (0.276"). Probability of overloads is small and breaker operation is expected only three or four times in a year. The spring will not be extended beyond  $L_2$  during service or installation.

For static application in an ambient environment, the material selected is ASTM A227.

1. Assume a clearance on O.D. of 10%:

$$\text{O.D.} = (0.9)(7) = 6.3 \text{ mm}$$

2. Assume  $S_2 = 700$  MPa uncorrected; let  $D = \text{O.D.} = 6.3$  mm

Calculate wire diameter  $d$ :

$$d = \sqrt[3]{\frac{2.55PD}{S}} = \sqrt[3]{\frac{(2.55)(30)(6.3)}{700}} = 0.88 \text{ mm}$$

$$\text{let } d = 0.9 \text{ mm}$$

Tensile strength taken from Figure 3-3, page 19, is 1790 MPa.

3. Calculate mean diameter  $D$  and coil index  $C$ :

$$D = \text{O.D.} - d = 6.3 - 0.9 = 5.4 \text{ mm,}$$

$$C = D/d = \frac{5.4}{0.9} = 6$$

4. Calculate mean stress at the extended length:

$$L_2 = 29.00 \text{ mm, } P_2 = 30 \text{ N}$$

$$S_2 = \frac{2.55 P_2 D}{d^3} K_{w1}$$

$$K_{w1} = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4(6) - 1}{4(6) - 4} + \frac{0.615}{6} = 1.25$$

$$S_2 = \frac{(2.55)(30)(5.4)(1.25)}{(0.90)^3} = 708 \text{ MPa or 40\% TS}$$

5. Calculate rate  $k$ :

$$k = \frac{P_2 - P_1}{L_2 - L_1} = \frac{30 - 17.5}{29 - 25} = 3.13 \text{ N/mm}$$

6. Calculate number of coils  $N_a$ :

$$N_a = \frac{Gd^4}{8D^3k} = \frac{(7.93 \times 10^4)(0.90)^4}{(8)(5.4)^3(3.13)} = 13.2$$

7. Calculate free length  $L_f$  and deflections  $f_1$  and  $f_2$ ; assume full twist loops:

$$L_f = 2(ID) + (N_a + 1)d = (2)(4.5) + (13.2 + 1)(0.9) = 21.78 \text{ mm}$$

$$f_1 = L_1 - L_f = 25.00 - 21.78 = 3.22 \text{ mm}$$

$$f_2 = L_2 - L_f = 29.00 - 21.78 = 7.22 \text{ mm}$$

8. Calculate initial tension  $P_1$  and uncorrected stress due to initial tension  $S_i$ :

$$P_1 = P_1 - kf_1 = 17.5 - (3.13)(3.22) = 7.42 \text{ N}$$

$$S_i = \frac{2.55 P_1 D}{d^3} = \frac{(2.55)(7.42)(5.4)}{(0.90)^3} = 140 \text{ MPa}$$

Referring to Figure 7-3, it can be seen that this is in the preferred range for initial stress for an index of 6.

9. Check stresses in the hooks:

Bending Stress:

$$S_A = \frac{16PD}{\pi d^3} K_1 + \frac{4P}{\pi d^2}; K_1 = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}; \text{ let } C_1 = C$$

$$K_1 = \frac{4(6)^2 - 6 - 1}{4(6)(6 - 1)} = 1.142$$

$$S_A = \frac{(16)(30)(5.4)(1.142)}{\pi(0.90)^3} + \frac{4(30)}{\pi(0.90)^2} = 1340 \text{ MPa or } 74.9\% \text{ TS}$$

Torsional stress, where  $R_2 = 2.70 \text{ mm}$ :

$$S_B = \frac{8PD}{\pi d^3} \left( \frac{4C_2 - 1}{4C_2 - 4} \right) \quad C_2 = \frac{2R_2}{d}$$

$$S_B = \frac{(8)(30)(5.4)}{\pi(0.90)^3} \left( \frac{4(6) - 1}{4(6) - 4} \right) = 651 \text{ MPa or } 36\% \text{ TS}$$

## 4.12 Final Specifications

- Free Length  $L_f$ : 21.78 mm (0.854") Reference
- Outside Diameter:  $6.3 \pm 0.10 \text{ mm}$  ( $0.248 \pm 0.004$ ")
- Wire Diameter  $d$ : 0.9 mm (0.035") Reference
- Initial Tension Load  $P_i$ : 7.45 N (1.68 lbf) Reference
- Extended Length  $L_1$ : 25.00 mm (0.984")
- Extended Length  $L_2$ : 29.00 mm (1.142")
- $P_1$  Load at  $L_1$ :  $17.5 \pm 2.0 \text{ N}$  ( $3.93 \pm 0.45 \text{ lbf}$ )
- $P_2$  Load at  $L_2$ :  $30 \pm 2.5 \text{ N}$  ( $6.74 \pm 0.55 \text{ lbf}$ )
- Final Design Stress  $S_2$ : 708 MPa (103,000 psi) 40% TS
- $N_a$ : 13.2 Coils

Refer to the load tolerances for helical extension springs (Table 7-5). Tolerance on load for  $P_1$  is  $\pm 11\%$ , which is less than the required  $\pm 15\%$ , and  $P_2$  is  $\pm 8\%$ , which is less than the required  $\pm 12\%$ .

## 5 GARTER SPRINGS

### 5.1 Introduction

Garter springs are helical extension or compression springs whose ends are connected so that each spring becomes a circle and exerts radial forces (Figure 8-1). Their primary application is in oil seals. Other uses include small motor belts, electrical connectors, and piston-ring expanders.

Design methods for garter springs are continuations of methods developed for helical compression and extension springs. Generally, garter springs are designed with as low a rate as possible to provide a nearly constant force on a seal as it wears.

### 5.2 Joint Design and Considerations

Garter spring ends may be fastened together by interlocking loops, staking one coil of the female end into the nub end; by soldering (for belt applications); or by screwing one end into the other. Associated Spring utilizes a special closure called Interlock<sup>®</sup>, which ensures positive and smooth connections (Figure 8-2).

Regardless of the connecting joint used, its strength must be such that the joint will not separate when the spring is extended to its maximum diameter.

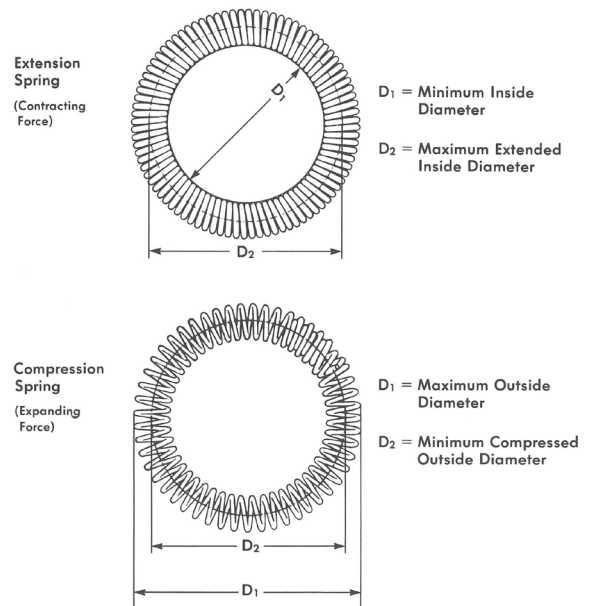
### 5.3 Spring Rate

Rate of an extension garter spring is expressed by the following equation:

$$(8-1) \quad k = \frac{P - P_1}{f} = \frac{Gd^5}{8D^3L_1} \text{ where } L_1 = \pi D_1 \approx N_a d$$

$L_1$  is the assembled length (not including the joint nib) and  $D$  = mean coil diameter. Index ( $D/d$ ) should be greater than four. If an index of less than four is required, consult Associated Spring engineers. Circumferential load is  $P_e = P_1 + kf$  for an extension-type garter spring, where  $P_1$  is initial tension and  $kf$  is the load due to deflection. On a compression garter spring, since  $P_1$  is equal to zero,  $P_c = kf$ .

Fig. 8-1. Extension and Compression Garter Springs.



### 5.4 Radial and Circumferential Load

Radial load  $P_r$  is defined as the load per unit length of circumference. The general equation for relating radial load  $P_r$  to circumferential load  $P$  is as follows:

(8-2)

$$P_r = \frac{2P}{D_2}$$

P is the load exerted by an extension spring at a deflection. Radial load can also be calculated directly using rate formulas where the deflection  $(f) = (D_2 - D_1)$ :

$$(8-3)$$

For extension type garter springs:

$$(8-4) \quad P_{re} = 2\pi k - \frac{2\pi D_1 k}{D_2} + \frac{2P_1}{D_2}$$

For compression type garter springs:

$$P_{rc} = \frac{2\pi D_1 k}{D_2} - 2\pi k$$

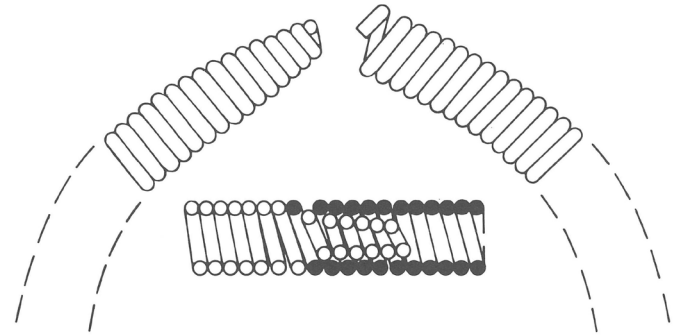
A compression garter spring is open wound without initial tension.

In most cases, radial load is calculated by measuring circumferential load and using  $v$  which relates them. A useful test procedure is to cut a spring 180° from the joint, then treat it as a normal extension spring.

## 5.5 Initial Tension and Stress

An ideal garter spring has a zero rate and applies a constant force as the seal wears. Since this condition is impossible, it is recommended that initial tension  $P_1$  be as close to circumferential load value as the design will permit. Uncorrected stress due to  $P_1$  should be limited to a range of 80 to 210 MPa (12,000 to 30,000 psi) with a preferred range of 110 to 150 MPa (16,000 to 22,000 psi) (See Figure 7-3). If  $P_1$  is too low, it is difficult to control the free length, which can cause the assembled diameter to vary. If  $P_1$  is too high, it may not be possible to coil the spring because adjacent coils tend to climb over each other. This nonuniformity in the body diameter causes erratic loads.

Fig. 8-2. Patented Interlock Method for Joining Garter Spring Ends.



## 5.6 Stress

Stress in a garter spring, based on circumferential load  $P$ , is derived from the equation used for compression and extension springs in static applications:

$$(8-5)$$

$$S = \frac{8PD}{\pi d^3} K_{W1}$$

Final torsional stress should be within 275 to 415 MPa (40,000 to 60,000 psi) for most garter springs.

### Tolerances

Coil diameter of the body will be held to  $\pm 0.13$  mm (0.005") unless otherwise specified.

Assembled Inside Diameter:	Tolerance	
	$\pm$ mm	in.
0 to 25 mm (1")	0.20	0.008
Over 25 (1") to 65.0 mm (2.5")	0.25	0.010
Over 65.0 (2.5") to 115 mm (4.5")	0.38	0.015
Over 115 mm (4.5"), consult Associated Spring	—	—

Load at a Linear Deflection of 5 mm (0.2"):

	$\pm$ N	ozf
ASTM A227 Steel:		
0 to 1.4 N (5 ozf)	0.14	0.5
1.4 N (5 ozf) and up	10%	10%
AISI Type 302 Stainless Steel:		
0 to 2.2 N (8 ozf)	0.22	0.8
2.2 N (8 ozf) and up	10%	10%

## 5.7 How to Specify

Only those characteristics that are critical to a garter spring's performance should be given with tolerances. Noncritical characteristics should be given as reference. This will enable the manufacturer to meet mandatory specifications,

such as loads and rates, without incurring extra costs. A suggested garter spring specification checklist for the designer is presented as a specification guide.

## 5.8 Garter Spring Design Example

The following information is given for a garter spring to be used in a radial shaft seal:

Given: Assembled free I.D. =  $D_1 = 21.00$  mm (0.827"). Extended installed I.D. =  $D_2 = 24.50$  mm (0.964"). Maximum O.D. (spring body) = 1.90 mm (0.075"). Desired radial load = 0.2,  $\pm .02$  N/mm (1.1,  $\pm .01$  lbf/in.) of circumference at  $D_2$ . Material is ASTM A227.

1. Assembled length  $L_1$ , extended length  $L_2$  and deflection  $f$  are calculated as follows:

$$L_1 = \pi D_1 = \pi(21.00) = 66.00 \text{ mm}$$

$$L_2 = \pi D_2 = \pi(24.50) = 77.00 \text{ mm}$$

$$f = L_2 - L_1 = 77.00 - 66.00 = 11.00 \text{ mm}$$

2. Circumferential load  $P$  is determined using the Equation 8-2 relating radial and axial loads:

$$P = \frac{P_r D_2}{2} = \frac{(0.2)(24.50)}{2} = 2.45 \text{ N}$$

3. Wire diameter is estimated assuming a design stress at the midpoint of the range, given in the previous section on stress, equal to 345 MPa,  $K_{w1} = 1.40$  assuming an index of 4 to start and a mean coil diameter:

$D$  = maximum outside coil diameter – tolerance on coil diameter

$$D = 1.90 - 0.13 = 1.77 \text{ mm}$$

$$d = \sqrt[3]{\frac{8PD}{\pi S} K_{w1}} = \sqrt[3]{\frac{(8)(2.45)(1.77)(1.40)}{\pi(345)}} = 0.36 \text{ mm}$$

Mean diameter is:  $D = 1.77 - 0.36 = 1.41$  mm. The index which is  $1.41/0.36 = 3.9$  is less than 4.

4. Assume a new wire diameter,  $d = 0.31$  mm,  $D = 1.77 - 0.31 = 1.46$  mm.

5. Recalculate rate and load due to deflection:

$$k = \frac{(7.93 \times 10^4)(0.31)^5}{(8)(1.46)^3(66)} = 0.138 \text{ N/mm}$$

$kf = (0.138)(11.00) = 1.52$  N. Because  $kf < P_e$ , the next step is to calculate the initial tension.

6. Initial tension  $P_1$  and stress  $S_1$  (uncorrected) are calculated as follows:

$$P_e = P - kf = 2.45 - 1.52 = 0.93 \text{ N}$$

$$S_1 = \frac{8(0.93)(1.46)}{\pi(0.31)^3} = 116 \text{ MPa}$$

The value of the stress due to initial tension is within the preferred range of 110 to 150 MPa, established in the paragraph on initial tension.

7. Final stress using  $K_{w1} = 1.33$  for an index of 4.7, is:

$$S = \frac{8PD}{\pi d^3} K_{w1} = \frac{(8)(2.45)(1.46)(1.33)}{\pi(0.31)^3} = 407 \text{ MPa}$$

This is within the guidelines for design stresses in the previous paragraph on stress.

8. Radial load is now checked.

$$P_{re} = 2\pi k - \frac{2\pi D_1 k}{D_2} + \frac{2P_1}{D_2}$$

$$P_{re} = 2\pi(0.138) - \frac{2\pi(21.0)(0.138)}{24.50} + \frac{(2)(0.93)}{24.50}$$

$$P_{re} = 0.2 \text{ N/mm}$$

**GARTER SPRING SPECIFICATION CHECKLIST**

(Fill in required data and apply tolerances only to critical dimensions.)

Suggested B/P Specifications:

Material \_\_\_\_\_  
 Assembled length \_\_\_\_\_mm(in.)  
 ± \_\_\_\_\_mm(in.)  
 Assembled I.D. (extension) or O.D. (compression)  
 \_\_\_\_\_mm(in.) ± \_\_\_\_\_mm(in.)  
 Installed I.D. or O.D. \_\_\_\_\_mm(in.) ± \_\_\_\_\_mm(in.)  
 Circumferential load required:  
 P \_\_\_\_\_N(lbf) ± \_\_\_\_\_N(lbf)  
 at L = \_\_\_\_\_mm(in.)  
 Rate \_\_\_\_\_N/mm(lbf/in.) Reference  
 Direction of coil: right \_\_\_\_\_left \_\_\_\_\_  
 optional \_\_\_\_\_  
 Type of end connection (if special) \_\_\_\_\_

Note: Tolerances may be specified either on assembled length or on assembled I.D., but not on both.

Reference or Calculated Data:

Wire diameter \_\_\_\_\_mm(in.),  
 O.D. body coil diameter \_\_\_\_\_mm(in.)  
 Number of coils: Active \_\_\_\_\_  
 Total (including nib) \_\_\_\_\_  
 Initial tension  $P_i$  \_\_\_\_\_N(lbf) Reference  
 Radial load  $P_r$  (calculated) \_\_\_\_\_N/mm (lbf/in.)  
 \_\_\_\_\_

Special Information:

Maximum operating temperature \_\_\_\_\_°C(°F)  
 Operating environment (water, heated oil etc.) \_\_\_\_\_

Finish requirements (finishes should be obtained by using preplated wire. See Section 3) \_\_\_\_\_

Rod or shaft diameter over which spring must expand without joint failure \_\_\_\_\_mm(in.)

## 5.9 Final Design Specifications

- Outside Coil Diameter: 1.77 ± 0.13 mm (0.070 ± 0.005")
- Material: ASTM A227
- Wire Diameter: 0.31 mm (0.012")
- Load P at 77.00 mm (3.031"): 2.45 N (0.55 lbf), ± 10%
- Assembled Length: 66.00 mm (2.598") Reference
- Spring Rate: 0.138 N/mm (0.79 lbf/in) Reference
- Calculated Initial Tension: 0.93 N (0.21 lbf) Reference
- Assembled I.D.: 21.0 ± 0.20 mm (0.827 ± 0.008")
- Final Design Stress ( $K_{W1}$  corrected): 407 MPa (59,000 psi)

- Stress Due to Initial Tension (uncorrected): 116 MPa (17,000 psi)

## 6 HELICAL TORSION SPRINGS

### 6.1 Introduction

Helical springs used to apply a torque or store rotational energy are commonly referred to as torsion springs. The two most common types are single- and double-bodied springs (Figure 9-1). Torsion springs are found in clothes pins, window shades, counterbalance mechanisms, ratchets, and various types of machine components. They are also used as couplings between concentric shafts, such as in a motor and pump assembly. Torsion springs are generally mounted around a shaft or arbor and must be supported at three or more points. Various kinds of ends are available to facilitate mounting.

Torsion springs are stressed in bending. Rectangular wire is more efficient in bending than round wire, but due to the premium cost of rectangular wire, round wire is preferred. If possible, a torsion spring should always be loaded in a direction that causes its body diameter to decrease. The residual forming stresses are favorable in this direction, but unfavorable when the spring is loaded in a direction that increases body diameter. Unless there are unfavorable residual stresses in the end bends, springmakers normally heat-treat these springs at a low temperature to stabilize the end positions rather than to fully stress-relieve them. If the direction of loading tends to increase body diameter, the springmaker should be advised to stress-relieve the springs.

The Associated Spring SPEC line contains many torsion spring designs using stainless steel and music wire, either left- or right-hand wound. These springs have tangent ends and are available for immediate delivery.

### 6.2 Number of Turns

The number of active turns in a helical torsion spring is equal to the number of body turns, plus a contribution from the ends. For straight torsion ends, this contribution is equal to one-third of the moment arms and is usually expressed as an equivalent number of turns:

(9-1)

$$N_e = \frac{L_1 + L_2}{3\pi D}$$

$L_1$  = length of the moment arm of the first end

$L_2$  = length of the moment arm of the second end

(9-2)

$$N_a = N_b + N_e$$

$N_b$  = number of body turns

### 6.3 Mean Diameter

Mean diameter is equal to I.D. plus O.D. divided by two. When the direction of loading ends to reduce the body diameter, the mean diameter changes with deflection according to:

(9-3)

$$D = \frac{D_1 N_b}{N_b + \theta}$$

where  $D_1$  is initial mean diameter and  $\theta$  is deflection in revolutions. Clearance must be maintained between the shaft or tube and spring at all times to prevent binding. The ideal shaft size is equal to, or slightly less than, 90% of the I.D. when the spring is fully deflected (minimum diameter). Shafts significantly smaller than 90% should be avoided to prevent buckling during large deflections.

## 6.4 Length

Most torsion springs are close-wound, with body length equal to the wire diameter multiplied by the number of turns plus one. When a spring is deflected in the direction that will reduce the coil diameter, body length increases according to:

(9-4)

$$L = d(N_b + 1 + \theta)$$

For applications that require minimum hysteresis, springs should be designed with space between adjacent coils to reduce frictional losses.

## 6.5 Spring Rate

Spring rate for helical round wire torsion springs is given by:

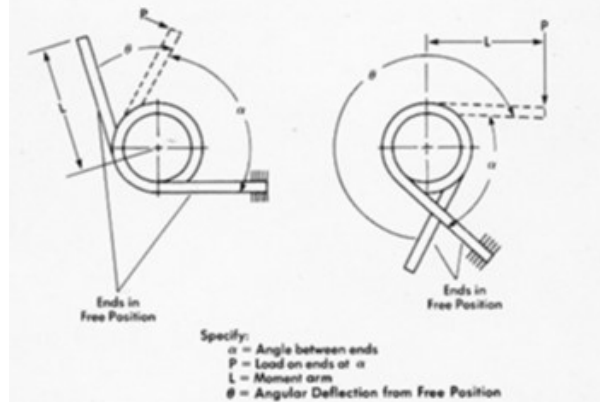
(9-5)

$$k = \frac{M}{\theta} = \frac{Ed^4}{10.8DN_a}$$

The 10.8 factor is greater than the theoretical factor of 10.2 to allow for friction between adjacent spring coils and between the spring body and the arbor. This factor is based on experience and has been found to be satisfactory.

Loads for torsion springs should be specified at a fixed angular position (Figure 9-1). Presently, there is no standard way to test loads for torsion springs. Consequently, in critical applications, it is advisable to contact Associated Spring engineers to establish a test method during prototype work.

Fig. 9-1. Specifying Load and Deflection Requirements for Torsion Springs.



## 6.6 Stress

Stress in torsion springs is due to bending, and for round wire is given by:

(9-6)

$$S = \frac{32M}{\pi d^3} K_B$$

During elastic deflection of a curved beam, the neutral axis shifts toward the center of curvature, causing higher stress at the inner surface than the outer. Wahl (Reference 3) has calculated the stress correction factor at the I.D. of a round wire torsion spring as:

(9-7)

$$K_{BID} = \frac{4C^2 - C - 1}{4C(C - 1)}$$

A convenient approximation for engineering calculations is:

(9-8)

$$K_{BID} = \frac{4C - 1}{4C - 4}$$

(9-9)

$$K_{BOD} = \frac{4C + 1}{4C + 4}$$

At low indexes, stress is significantly higher on the inner surface than the outer. These factors are useful to determine the stress range for cyclic applications and the set point for fully stress-relieved springs in static applications. A stress correction factor of 1 is recommended to determine the set point of springs that have

favorable residual stresses induced by yielding during forming. Yielding results in a more uniform stress distribution over the round cross-section. Therefore, the actual stress correction factor approaches the recommended value of 1.

## 6.7 End Configurations

Some of the more common end configurations available are shown in Figure 9-2. Special configurations are available on request. In designing ends, it is important to recall that bends, loaded to decrease their radius of curvature, have favorable residual stresses. They can operate at higher applied stress levels than bends that increase the radius by loading. Frequently, spring performance is limited because the sharply bent ends have greater stress than the body. Equation 9-6 is generally employed to determine maximum bending stress in the ends. Torsion springs are subject to surging and resonance phenomena. The natural frequency  $n$  for a torsion spring with one end fixed is:

$$n = \frac{1.26 \times 10^3 d}{\pi D^2 N_a} \sqrt{\frac{Eg}{\rho}}; \text{ for steel} = \frac{2 \times 10^5 d}{D^2 N_a} \text{ metric} \quad (9-10)$$

$$n = \frac{d}{8\pi D^2 N_a} \sqrt{\frac{Eg}{\rho}}; \text{ for steel} = \frac{8040d}{D^2 N_a} \text{ English}$$

And with both ends fixed:

$$n = \frac{2.5 \times 10^3}{\pi D^2 N_a} d \sqrt{\frac{Eg}{\rho}}; \text{ for steel} = \frac{4 \times 10^5 d}{D^2 N_a} \text{ metric} \quad (9-11)$$

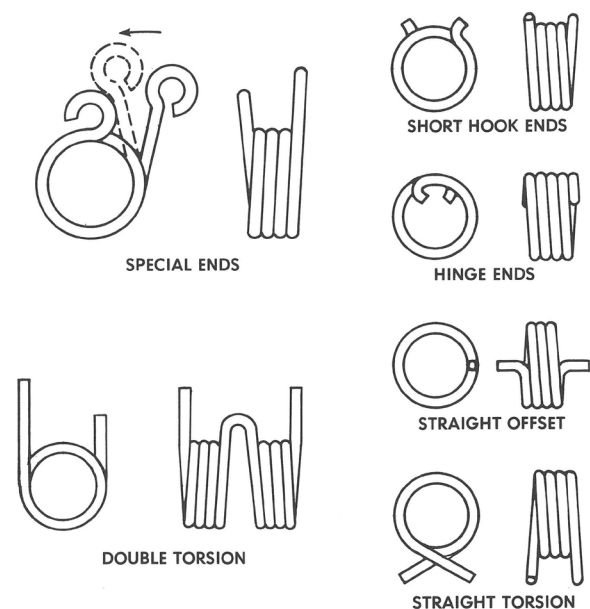
$$n = \frac{d}{4\pi D^2 N_a} \sqrt{\frac{Eg}{\rho}}; \text{ for steel} = \frac{16080d}{D^2 N_a} \text{ English}$$

To avoid or minimize resonance phenomena, the natural frequency must be much greater than the operating frequency and/or the spring should contain initial tension.

## 6.8 Choice of Operating Stress—Static

Recommended maximum operating stresses for static applications are given as percentages of tensile strength in Table 9-1. For spring bodies or ends loaded in a direction that increases the radius of curvature, stress levels in the “stress-relieved” column are most appropriate. These stresses should be calculated using the appropriate  $K_B$  stress correction factor (Equation 9-8 or 9-9). When the outer surface is in tension, springs with a low index usually yield at the inner surface, while those with a high index may yield at the outer surface. For springs not stress-relieved and loaded in a direction that decreases the radius of curvature, the stress levels recommended for springs with favorable residual stress are appropriate. No stress correction factor is used since the spring has yielded.

Fig. 9-2. Common Helical Torsion Spring End Configurations.



## 6.9 Choice of Operating Stress—Cyclic

Maximum allowed operating stresses for cyclic applications are presented in Table 9-2 as percentages of tensile strength. All stresses are assumed to be calculated with the appropriate  $K_B$  correction factor. This information can be used to estimate fatigue lives at other stress ranges by methods discussed previously (Section 4). Frequently, bending stresses are higher in the ends than in the body. In this situation, bear in mind that during forming of sharp bends, the wire may be stretched or marked, resulting in stress concentrations that reduce design stress levels below those recommended. Because of friction, the point of contact between torsion end and arbor is often the highest stressed area.

## 6.10 Double Torsion Springs

Double-bodied torsion springs are designed using the same methods as for single-bodied torsion springs. The rate for a double-bodied torsion spring is equal to the sum of the rates for each component. For the same wire diameter, coil diameter, and wire length, double-bodied torsion springs have rates four times those of single-bodied types. Double-bodied torsion springs should be designed so they are coiled out from the center rather than in from the ends (Figure 9-3).

## 6.11 Rectangular Wire

Rectangular wire torsion springs have higher energy storage capacities than similar round wire springs. The general comments on round wire torsion springs apply to springs with rectangular wire.

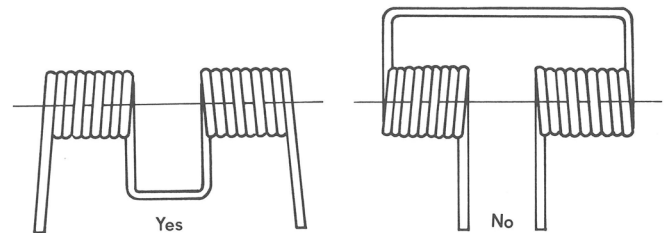
**Table 9-1.** Maximum Recommended Bending Stresses for Helical Torsion Springs in Static Applications.

Material	Percent of Tensile Strength	
	Stress-Relieved (1) ( $K_B$ Corrected)	With Favorable Residual Stress (2) (No Correction Factor)
Patented and Cold Drawn	80	100
Hardened and Tempered Carbon and Low Alloy Steels	85	100
Austenitic Stainless Steels and Non-Ferrous Alloys	60	80

(1) Also for springs without residual stresses.

(2) Springs that have not been stress-relieved and which have bodies and ends loaded in a direction that decreases the radius of curvature.

**Fig. 9-3.** Preferred Winding for Double-Bodied Torsion Springs.



In producing springs from rectangular wire, the wire cross-section distorts and becomes “keystoned” (Figure 9-4). The wire axial dimension  $b_1$  can be estimated from: (9-12)

$$b_1 = b \left( \frac{C + 0.5}{C} \right)$$

When axial length is critical, keystone-shaped wire can be purchased. This wire will have a near-rectangular shape after coiling. The rate equation is: (9-13)

$$k = M / \theta = \frac{Ebt^3}{6.6 DN_a}$$

and the stress equation is: (9-14)

$$S = \frac{6M}{bt^2} K_B$$

These equations are for springs wound either on edge or on flat (Figure 9-4). Stress correction

factor  $K_B$  is slightly lower than for round wire and can approximated by:

$$(9-15) \quad K_{BID} = \frac{4C}{4C - 3}$$

$$(9-16) \quad K_{BOD} = \frac{4C}{4C + 3}$$

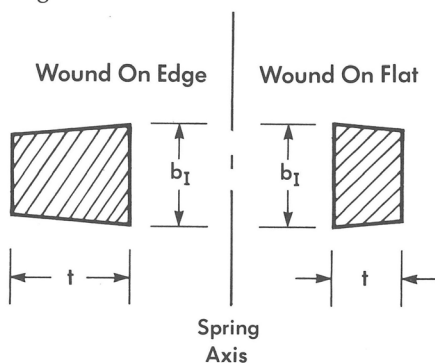
Sharp corners on rectangular wire cause stress concentrations and should be avoided, while generous corner radii of rolled wire reduce the wire cross-sections sufficiently to lower the rate.

**Table 9-2. Maximum Recommended Bending Stresses ( $K_B$  Corrected) for Helical Torsion Springs in Cyclic Applications.**

Fatigue Life (cycles)	Percent of Tensile Strength			
	ASTM A228 and Type 302 Stainless Steel		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened*	Not Shot-Peened	Shot-Peened*
10 <sup>5</sup>	53	62	55	64
10 <sup>6</sup>	50	60	53	62

This information is based on the following conditions: no surging, springs are in the "as-stress-relieved" condition  
 \*Not always possible.

**Fig. 9-4. Keystoned Cross Sections of Springs Wound on Edge and Flat**



### 6.12 Tolerances

Coil diameter and end position tolerances for helical torsion springs are presented in Tables 9-3 and 9-4 respectively. These tolerances should serve as guidelines, applied only to those dimensions critical to spring function. Closer tolerances are available upon request.

### 6.13 How to Specify

The accompanying helical torsion spring specification checklist is suggested as a guide to designers and a vehicle for improved communications between designer and springmaker.

### 6.14 Design Example

Design a cabinet door hinge torsion spring to hold the door closed by exerting a torque  $M = 55 \text{ N mm}$  (0.49 lb-in) at  $\alpha = 90^\circ$  (see Figure 9-1) between ends, each with a moment arm of 19 mm (0.748") and tangent to the body. When the door is fully opened, the spring deflects through additional overtravel of  $120^\circ$  or  $1/3$  revolution. Maximum spring length is 13 mm (0.512"), and the spring operates over 6.0 mm (0.236") diameter arbor  $D_a$ . Required life is 5,000 cycles. Use oil-tempered spring steel wire ASTM A229.



1. Assume a deflection  $\theta_1$ , from free to first loaded position is:

$$\theta_1 = \Delta\theta = 1/3 \text{ revolution}$$

Calculate  $\theta_2$ , the angular deflection to second loaded position:

$$\theta_2 = \theta_1 + \theta = 2/3 \text{ revolution}$$

2. Calculate spring rate  $k$ :

$$k = \frac{M}{\theta} = \frac{55}{1/3} = 165 \text{ N} \cdot \text{mm/revolution}$$

Calculate torque  $M_2$  at  $\theta_2$  deflection:

$$M_2 = M_1 + k(\Delta\theta) = 55 + 165(1/3) = 110 \text{ N} \cdot \text{mm}$$

3. Assume stress  $S_2$  (at  $M_2$ ) to be equal to 1400 MPa and solve for:

$$d = \sqrt[3]{\frac{32M_2}{\pi S_2}} = \sqrt[3]{\frac{32(110)}{\pi 1400}} = 0.9 \text{ mm}$$

4. Assuming the clearance between the arbor and spring inner surface to be 25% of arbor diameter, calculate mean diameter:

$$D = 1.25 \times D_a + d = 1.25 \times 6.0 + 0.9 = 8.4 \text{ mm diameter}$$

5. Calculate number of coils  $N_a$  from:

$$N_a = \frac{Ed^4}{10.8Dk} = \frac{(20.7 \times 10^4)(0.9)^4}{10.8(8.4)(165)} = 9.1$$

$$\text{where } N_e = \frac{L_1 + L_2}{3\pi D} = \frac{38}{3\pi(8.4)} = 0.5$$

is a correction for ends. Therefore  $N_b = 8.6$  coils

## 6.15 Final Design Specifications

- Material: ASTM A229
- Outside Coil Diameter:  $9.0 \pm 0.09 \text{ mm}$  ( $0.354 \pm 0.0035$ " )
- Wire Diameter,  $d$ :  $0.9 \text{ mm}$  ( $0.035$ " ) Reference
- $M_1$  at  $a_1$  position:  $55 \pm 5.5 \text{ N} \cdot \text{mm}$  ( $0.49 \pm 0.05$  lbf-in)

6. Adjust  $N_b$  so that partial coil agrees with the desired position of ends when spring is unloaded. Because  $90^\circ + 120^\circ$  is greater than  $180^\circ$ , the desired partial coil is:

$$\frac{540 - (90 + 360 \times 1/3)}{360} = 0.9$$

Therefore,  $N_b = 8.9$  and the adjusted mean diameter is:

$$D = \frac{(20.7 \times 10^4)(0.9)^4}{10.8(8.9 + 0.5)(165)} = 8.1 \text{ mm}$$

7. Calculate length of spring in loaded position:

$$L = (N_b + 1 + \theta_2)d = (8.90 + 0.67 + 1) \times 0.9$$

$$L = 9.5 \text{ mm}$$

8.  $9.5 < 0.9 \times 13 = 11.7 \text{ mm}$ , therefore the length is satisfactory.

9. Calculate clearance  $\Delta$ :

$$\Delta = \frac{DN_b}{N_b + \theta_2} - d - D_a = \frac{(8.9)(8.1)}{(8.9 + 0.66)} - 0.9 - 6.0$$

$$\Delta = 0.641 \text{ mm}$$

10. Check whether the clearance is safe:

$$0.64 \text{ mm} > 0.1 \times 6.0 = 0.60 \text{ mm}$$

Therefore the clearance is satisfactory.

11. Check bending stress level for static application: TS of 0.9 mm wire is 1870 MPa (271,000 psi) (Figure 3-3, page 19)

$$S_2 = \frac{1400}{1870} 100 \approx 75\% \text{ TS} < 100\% \text{ (Table 9-1)}$$

Therefore the stress level is satisfactory.

lbf-in)

- $M_2$  at  $a_2$  position:  $110 \text{ N} \cdot \text{mm}$  ( $0.98 \text{ lbf-in}$ ) Reference
- $N_b$ : 8.9 coils  $\pm 10^\circ$
- Ends: Straight Torsion
- Final Design Stress: 1400 MPa (203,000 psi) or 75% TS

**HELICAL TORSION SPRING SPECIFICATION CHECKLIST**

(Fill in required data only)

Material \_\_\_\_\_

**Working Conditions:**

To work in \_\_\_\_\_mm(in.) diameter hole

To work over \_\_\_\_\_mm(in.) diameter shaft

Torque \_\_\_\_\_ N·mm(lb-in.) , ± \_\_\_\_\_ N·mm(lb-in.)  
when angle between ends is \_\_\_\_\_degrees

Torque \_\_\_\_\_ N·mm(lb-in.) , ± \_\_\_\_\_ N·mm(lb-in.)  
when angle between ends is \_\_\_\_\_degrees

Axial space required \_\_\_\_\_mm(in.)

Direction of coil (right or left-hand) \_\_\_\_\_

Maximum wound position \_\_\_\_\_revolutions or degrees

**Reference Data:**

Wire diameter \_\_\_\_\_mm(in.)

Mean diameter \_\_\_\_\_mm(in.)

Number of coils \_\_\_\_\_

Rate \_\_\_\_\_ N·mm /revolution (lb-in./revolution)

**Special Information:**

Finish \_\_\_\_\_

Loading (cyclic, impact, static, other) \_\_\_\_\_

Required life \_\_\_\_\_ cycles

Required reliability (see Section 4) \_\_\_\_\_

Operating deflection range \_\_\_\_\_ revolutions

Maximum operating temperature \_\_\_\_\_ °C(°F)

Operating environment \_\_\_\_\_

*Table 9-3. Commercial Tolerances for Torsion Spring Coil Diameters.*

Wire Diameter mm (in.)	Tolerance: ±mm (in.)						
	Spring Index D/d						
	4	6	8	10	12	14	16
0.38 (0.015)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.08 (0.003)	0.10 (0.004)
0.58 (0.023)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.13 (0.005)	0.15 (0.006)
0.89 (0.035)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.23 (0.009)
1.30 (0.051)	0.05 (0.002)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.20 (0.008)	0.25 (0.010)	0.31 (0.012)
1.93 (0.076)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.23 (0.009)	0.31 (0.012)	0.38 (0.015)	0.46 (0.018)
2.90 (0.114)	0.10 (0.004)	0.18 (0.007)	0.25 (0.010)	0.33 (0.013)	0.46 (0.018)	0.56 (0.022)	0.71 (0.028)
4.37 (0.172)	0.15 (0.006)	0.25 (0.010)	0.33 (0.013)	0.51 (0.020)	0.69 (0.027)	0.86 (0.034)	1.07 (0.042)
6.35 (0.250)	0.20 (0.008)	0.36 (0.014)	0.56 (0.022)	0.76 (0.030)	1.02 (0.040)	1.27 (0.050)	1.52 (0.060)

*Table 9-4. End Position Tolerances (for D/d Ratios up to and Including 16).*

Total Coils	Tolerance: ± Degrees*
Up to 3	8
Over 3-10	10
Over 10-20	15
Over 20-30	20
Over 30	25

\*Closer tolerances available on request.



## 7 RETAINING RINGS

### 7.1 Introduction

Retaining rings, also referred to as snap rings, primarily provide shoulders to locate or retain parts on shafts or in cylinders. Advantages of retaining rings are their economy of manufacture, ease of assembly, and accuracy of positioning as compared to threaded fasteners.

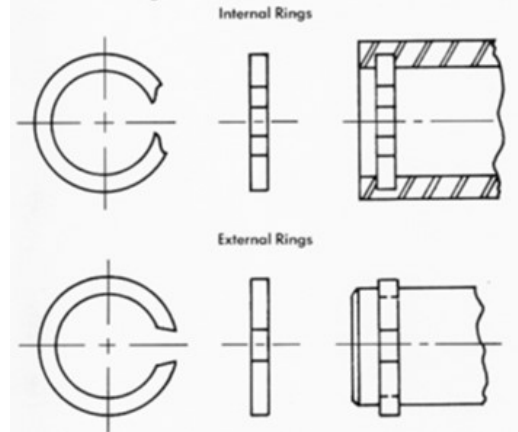
The two major categories of retaining rings are internal and external (Figure 10-1). External rings are assembled over a shaft, while internal rings are assembled in a cylinder. Some external rings are installed over the shaft radially and require a larger gap than rings that are installed axially.

A primary design requirement is that a retaining ring stay in place throughout an assembly lifetime. Normally, thrust loads are low. Provided the ring is loaded uniformly, thrust loads are not limited by material strength. After coiling, the material thickness near the inner surface of the ring is greater than that of the original material (see subsection titled "Rectangular Wire" in Section 8). This change in thickness must be considered when selecting the wire and groove size. If required, an increase in thickness can be reduced by secondary operations.

### 7.2 End Configurations

Several common end configurations for retaining rings made from rectangular wire are shown in Figure 10-2. Selection of end configurations for round wire retaining rings is limited to inside and outside bends. Sharp bends in rectangular wire will cause it to keystone severely and may require grinding to hold a specified maximum thickness.

Fig. 10-1. Typical Internal and External Retaining Rings.



### 7.3 Loads and Deflection

The maximum bending load on retaining rings usually occurs at a location opposite the gap during assembly and disassembly. Service loads are generally low. Loads and deflections are the same whether a ring is loaded externally or internally, as shown in the following equations:

$$(10-1) \quad P_{f1} = P_{fE} = \frac{Ed^4f}{4D^3}; \text{round wire, } 90^\circ \text{ from center of gap}$$

$$(10-2) \quad P_{f1} = P_{fE} = \frac{4Ebt^3f}{3\pi D^3}; \text{rectangular wire, } 90^\circ \text{ from center of gap}$$

$$(10-3)$$

$$(10-4) \quad P_{F1} = P_{FE} = \frac{Ed^4F}{24D^3}; \text{round wire, gap}$$

$$P_{F1} = P_{FE} = \frac{4Ebt^3F}{18\pi D^3}; \text{rectangular wire, gap}$$

$f$  = total deflection at the middle of the ring.

$F$  = total deflection at the gap (see Figures 10-3 and 10-4).

Note that  $b$  is the axial dimension of the rectangular wire and  $t$  is the radial dimension.

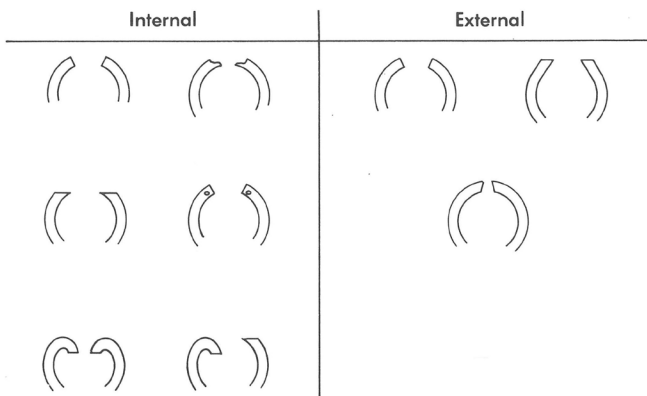
### 7.4 Stress

Retaining ring bending stress depends upon direction of loading and is determined from equations in Table 10-1.

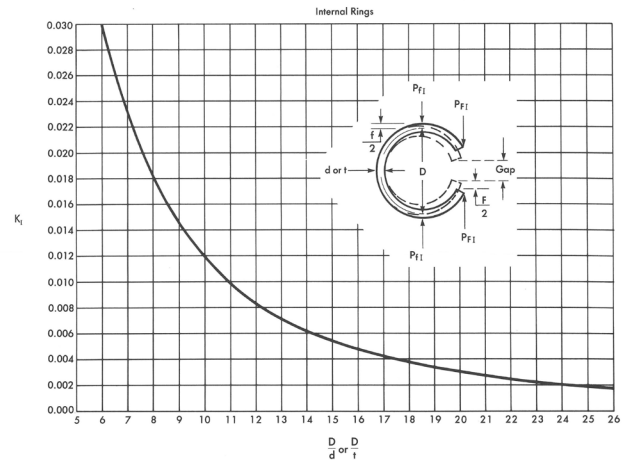
### 7.5 Choice of Stress Level

Most retaining rings are made from AISI 1065 steel at HRC 40 to 50 hardness level. Maximum allowable design stresses are shown in Table 10-2. Retaining rings fabricated by hardening and tempering after forming have no residual stresses; retaining rings fabricated from pre-tempered or hard-drawn wire contain residual stresses. If set is permitted in an application, stress levels can be increased by 10 to 20 percent depending on the amount of set permitted. If these higher stress levels must be used, consult Associated Spring engineers.

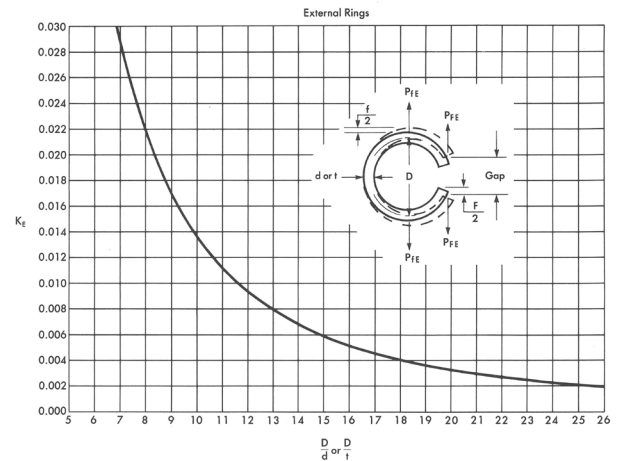
*Fig. 10-2. Typical End Configurations for Internal and External Retaining Rings.*



*Fig. 10-3. Stress Correction Factors for Retaining Rings in Contraction Applications.*



*Fig. 10-4. Stress Correction Factors for Retaining Rings in Expansion Applications.*



### 7.6 Tolerances

Standard tolerances for wire retaining rings are presented in Table 10-3. When measuring ring diameter, the scale must be held at right angles to the gap center line. Closer tolerances are available on request.

### 7.7 How to Specify

The accompanying retaining ring specification checklist will aid an occasional designer of retaining rings in considering all pertinent specifications.

### 7.8 Retaining Ring Design Example

Design internal retaining rings of O.D. = 60.00 mm (2.362") and wire cross-sectional dimensions before coiling of  $t = 3\text{mm}$  (0.118") and  $b = 1.5\text{mm}$  (0.059"). They are to be used in an application where bore is 55.25 mm (2.175"), groove diameter is 58.25 mm (2.293"), and width is 1.60 mm (0.063"). Material is (cold-drawn) ASTM A227 and finished ring hardness ranges from HRC 46 to 48, with a corresponding minimum tensile strength of 1525 MPa (221,000 psi). The rings must not set when assembled.

1. Calculate stress due to deflection  $f$  to assemble a ring:  
 $f = \text{O.D. of ring minus the bore diameter}$   
 $f = 60.00 - 55.25 = 4.75 \text{ mm}$

Index  $C = \frac{D}{t} = 19$ . From Figure 10-3, the correction factor  $K_I$  for stress calculations is 0.0035. Stress is calculated as follows:

$$S_I = \frac{fEK_I}{t} = \frac{(4.75)(20.7 \times 10^4)(0.0035)}{3}$$

$$S_I = 1147 \text{ MPa or } 75\% \text{ TS}$$

Referring to Table 10-2, it can be seen that for patented and cold-drawn materials not stress-relieved, stresses in bending for infrequent deflections can be up to 100% TS without setting.

2. Ring material when coiled will keystone to some degree. To verify that a groove width is large enough, the finished dimension of the cross section is checked:

$$b_1 = b \left( \frac{C + 0.5}{C} \right) = (1.5) \left( \frac{19 + 0.5}{19} \right) = 1.54 \text{ mm}$$

3. Load and stress at the assembled position are calculated as follows:

$$f = \text{O.D. of ring minus installed groove diameter} = 60.00 - 58.25$$

$$f = 1.75 \text{ mm}$$

$$P = \frac{4Efbt^3}{3\pi D^3} = \frac{4(20.7 \times 10^4)(1.75)(1.5)(3)^3}{3\pi(57)^3} = 33.6 \text{ N}$$

$$S = \frac{3\pi PD^3 K_I}{4bt^4} = \frac{3\pi(33.6)(57)^3(0.0035)}{(4)(1.5)(3)^4}$$

$$S = 422 \text{ MPa or } 28\% \text{ TS}$$

### 7.9 Final Design Specifications

- Material: ASTM A227
- Thickness: 1.5 mm (0.059")
- Width: 3.0 mm (0.118")
- Wound on Edge
- O.D. in Free Position: 60.00 +0.78, -0.00 mm (2.126 +0.031, -0.000")
- Stress During Assembly: 1147 MPa (166,000 psi)
- Load in Installed Position: 3.36 N (7.6 lbf) Reference

**Table 10-1. Stress Equations for Retaining Rings.**

Round Wire	Rectangular Wire	Deflection & Load
$S_I = \frac{fEK_I}{d} \text{ or } \frac{4PD^3 K_I}{d^5}$	$S_I = \frac{fEK_I}{t} \text{ or } \frac{3\pi PD^3 K_I}{4bt^4}$	90° from center of gap
$S_I = \frac{FEK_I}{3d} \text{ or } \frac{8PD^3 K_I}{d^5}$	$S_I = \frac{FEK_I}{3t} \text{ or } \frac{3\pi PD^3 K_I}{2bt^4}$	At the gap
$S_E = \frac{fEK_E}{d} \text{ or } \frac{4PD^3 K_E}{d^5}$	$S_E = \frac{fEK_E}{t} \text{ or } \frac{3\pi PD^3 K_E}{4bt^4}$	90° from center of gap
$S_E = \frac{FEK_E}{3d} \text{ or } \frac{8PD^3 K_E}{d^5}$	$S_E = \frac{FEK_E}{3t} \text{ or } \frac{3\pi PD^3 K_E}{2bt^4}$	At the gap

$d$  = wire diameter.  
 $t$  = radial dimension of rectangular cross-section.  
 $b$  = axial dimension.  
 $f$  = deflection 90° from center of gap.  
 $F$  = deflection at the gap of the ring.  
 $P_{fI} = P_{fE}$  is the load 90° from center of gap (Figure 10-3).  
 $P_{fI} = P_{fE}$  is the load at the gap of the ring (Figure 10-4).  
 $S_I$  = stress for internal rings.  
 $S_E$  = stress for external rings.  
 $K_I$  = correction factor given in Figure 10-3.  
 $K_E$  = correction factor given in Figure 10-4.

**Table 10-2. Maximum Design Stress Levels for Carbon Steel Retaining Rings in Static Applications.**

Type	Percent of Tensile Strength	
	Stress-Relieved	With Residual Stress
External	80	—
Internal	80	100

Internal retaining rings contain favorable residual stresses as formed and are not usually stress-relieved. External rings contain unfavorable residual stresses as formed and are usually stress-relieved prior to use.



*Table 10-3. Free Dimension Tolerances.*

Inside Diameter of Ring, Free mm (in.)	Inside Diameter		Gap mm (in.)
	External Rings mm (in.)	Internal Rings mm (in.)	
under 32 (1.25)	+0.00 (0.000) -0.38 (0.015)	+0.38 (0.015) -0.00 (0.000)	±0.78 (0.031)
32-76 (1.25-3)	+0.00 (0.000) -0.78 (0.031)	+0.78 (0.031) -0.00 (0.000)	±1.58 (0.062)
76-127 (3-5)	+0.00 (0.000) -2 (0.079)	+2.00 (0.079) -0.00 (0.000)	±2.36 (0.093)
127 (5) and over	+0.00 (0.000) -3.17 (0.125)	+3.17 (0.125) -0.00 (0.000)	± 3.17 (0.125)

**RETAINING RING SPECIFICATION CHECKLIST**

(Fill in required data only)

Material: Unless Otherwise Specified: AISI 1065

Working Conditions:

External \_\_\_\_\_ Internal \_\_\_\_\_  
 To work in \_\_\_\_mm(in.) diameter, \_\_\_\_mm(in.) groove width  
 Load \_\_\_\_\_N(lbf) at \_\_\_\_\_mm(in.)  
 Type of ends \_\_\_\_\_  
 Method of assembly: axial \_\_\_\_\_ radial \_\_\_\_\_  
 Maximum gap during assembly \_\_\_\_\_mm(in.)

Special Information:

Maximum operating temperature \_\_\_\_\_°C(°F)  
 Operating environment \_\_\_\_\_  
 Finish \_\_\_\_\_

Design Data (Reference):

Material size \_\_\_\_\_mm(in.)  
 Diameter of ring in free position \_\_\_\_\_mm(in.)



## 8 BELLEVILLE SPRING WASHERS

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### 8.1 Introduction

Belleville washers, Figure 11-1, also known as coned-disc springs and Belleville disc springs, were patented in France by Julien F. Belleville in 1867. When a load is applied, the washer tends to flatten causing radial and circumferential strains. This elastic deformation constitutes the spring action.

Belleville washers are used in two broad categories of applications. In one case, they provide very high loads at small deflections. Examples of these applications are stripper springs used in metal stamping dies and stacks of Belleville washers used in recoil mechanisms and pressure relief valves. The other category utilizes the special nonlinear load deflection characteristics of Belleville washers—particularly those with a constant load portion. In these applications, they maintain a constant force regardless of dimensional variations due to wear, temperature changes, or tolerances. Typical applications include packing seals, lathe live centers, and clutches.

The two types of performance depend on their height-to-thickness ratios. Typical load deflection curves for various ratios are shown in Figure 11-2. Note that the curve for low  $h/t$  ratios is nearly a straight line. At  $h/t = 1.41$ , the curve is nearly constant, with respect to load, for approximately the last 50% of deflection before flat and the first 50% deflection past flat. Above  $h/t = 1.41$ , the load decreases after reaching a peak. When  $h/t$  is greater than 2.83 (at some point beyond flat), the washer will snap through and will require a force to restore it to its free position.

Design formulas are complex and present

a difficult challenge to an occasional spring designer. Associated Spring RAYMOND has hundreds of pre-engineered Belleville washer designs in the SPEC product line. Selecting a SPEC Belleville washer not only saves design time but also avoids tooling costs, improves delivery, and is generally more cost-effective than a custom-designed Belleville washer.

### 8.2 Design Considerations

The method of mounting and loading affects performance and design of Belleville washers. If a washer is mounted on a flat plate, its useful range of deflection for accurate loads is from 15% to 85%  $h$ . Deflections beyond 85%  $h$  are possible, but loads will be considerably higher than calculated. Loads should be applied uniformly around the inner and outer edges. Generous radii on the corners cause a decrease in the amount of active material, shorten the moment arm, and increase the rate of Belleville washers. Sharp radii cause stress concentrations and severely reduce fatigue life.

Some Belleville washers must deliver precise loads near the flat position or be deflected beyond the flat position. In such cases, a method of mounting is recommended that positions the washer, applies load uniformly around its circumference, maintains a relatively constant amount of active material, and provides a positive stop to prevent overtravel. One method is illustrated in Figure 11-3. Here, the rate will increase when the washer is deflected beyond flat because the moment arm decreases.

Stress is not distributed uniformly in Belleville washers. The highest stress is at the top inner edge and is compressive (Figure 11-4). Highest

tensile stresses are generated at both bottom corners.  $S_{T2}$  is higher than  $S_{T1}$  for most Bellevilles with  $h/t$  ratios greater than 0.6 (Figure 11-5). In cyclic applications, it is good practice to calculate stress ranges at both locations. Belleville washers should be designed with stresses low enough to prevent setting if the washer is accidentally compressed to the flat position.

### 8.3 Design Calculations

Design equations are taken from the mathematical analysis by Almen and Laszlo and are: (Reference 5)

$$(11-1) \quad P = \frac{Ef}{(1-\mu^2)Ma^2} [(h-f)(h-f/2)t + t^3]$$

$$(11-2) \quad P_f = \frac{Eht^3}{(1-\mu^2)Ma^2}$$

$$(11-3) \quad S_c = \frac{-Ef}{(1-\mu^2)Ma^2} [C_1(h-f/2) + C_2t]$$

$$(11-4) \quad S_{T1} = \frac{Ef}{(1-\mu^2)Ma^2} [C_2t - C_1(h-f/2)]$$

$$(11-5) \quad S_{T2} = \frac{Ef}{(1-\mu^2)a^2} [T_1(h-f/2) + T_2t]$$

- $A = O.D./2$
- $C_1 =$  compressive stress constant (see Figure 11-6)
- $C_2 =$  compressive stress constant (see Figure 11-6)
- $h =$  inside height
- $H =$  overall height
- $M =$  constant (Figure 11-6)
- $PF =$  load at flat position
- $R = O.D./I.D.$
- $S_c =$  compressive stress at convex I.D. corner

- $S_{T1} =$  tensile stress at concave I.D. corner
- $S_{T2} =$  tensile stress at concave O.D. corner
- $T_1 =$  tensile stress constant (see Figure 11-7)
- $T_2 =$  tensile stress constant (see Figure 11-7)

In deriving these equations, it was assumed that angular deflection of the cross section is relatively small, the cross-section is not distorted during deflection, and load is distributed uniformly around the circumference.

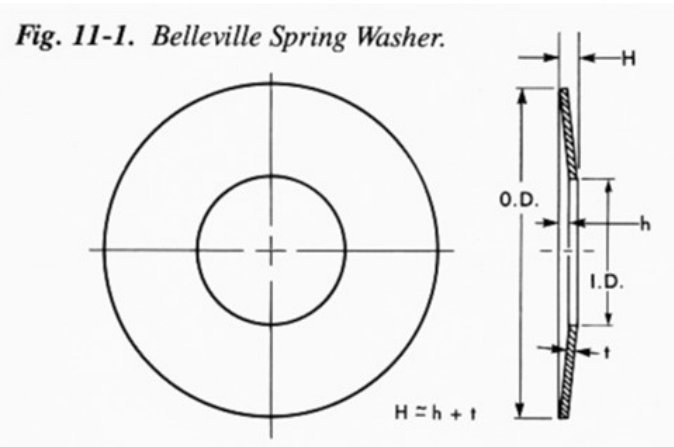
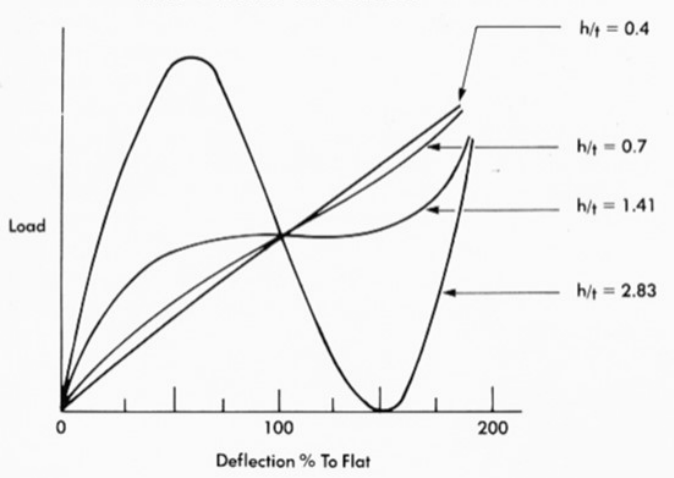
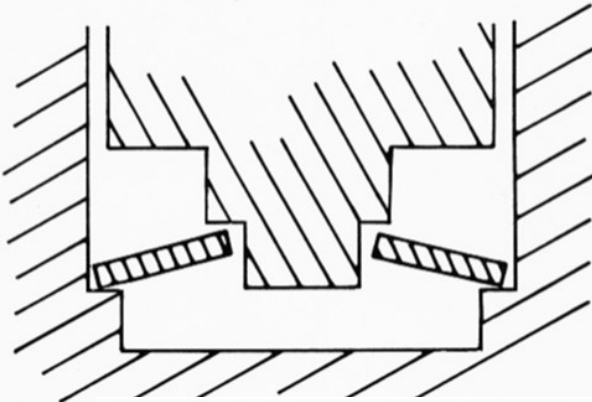


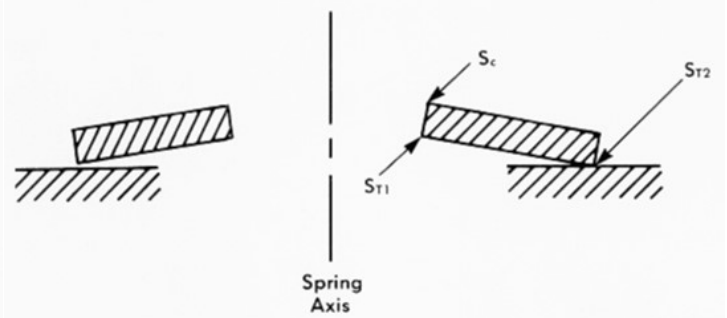
Fig. 11-2. Load Deflection Curves for Belleville Washers with Various  $h/t$  Ratios.



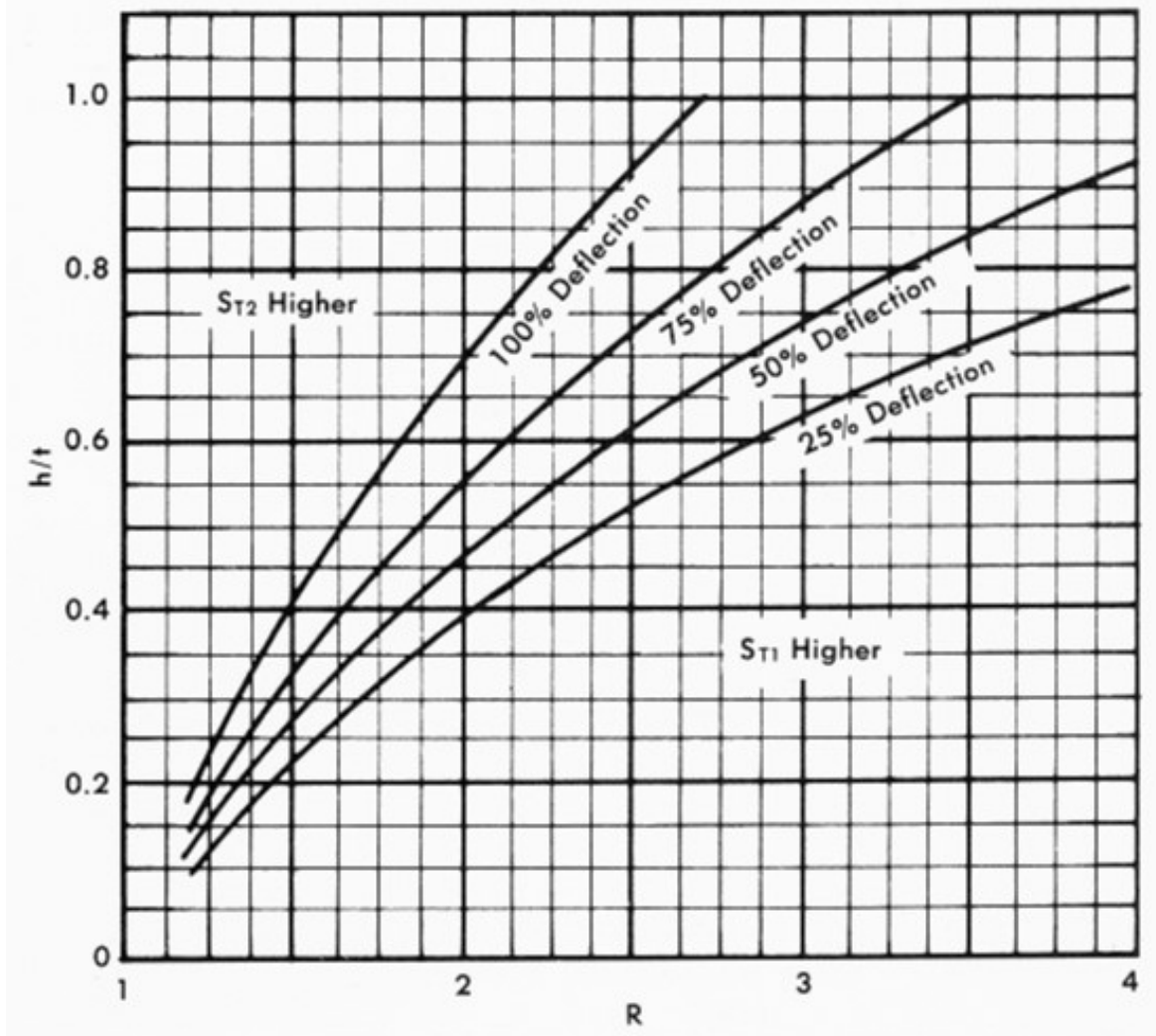
**Fig. 11-3.** Mounting a Belleville Washer for Deflection Past the Flat Position.



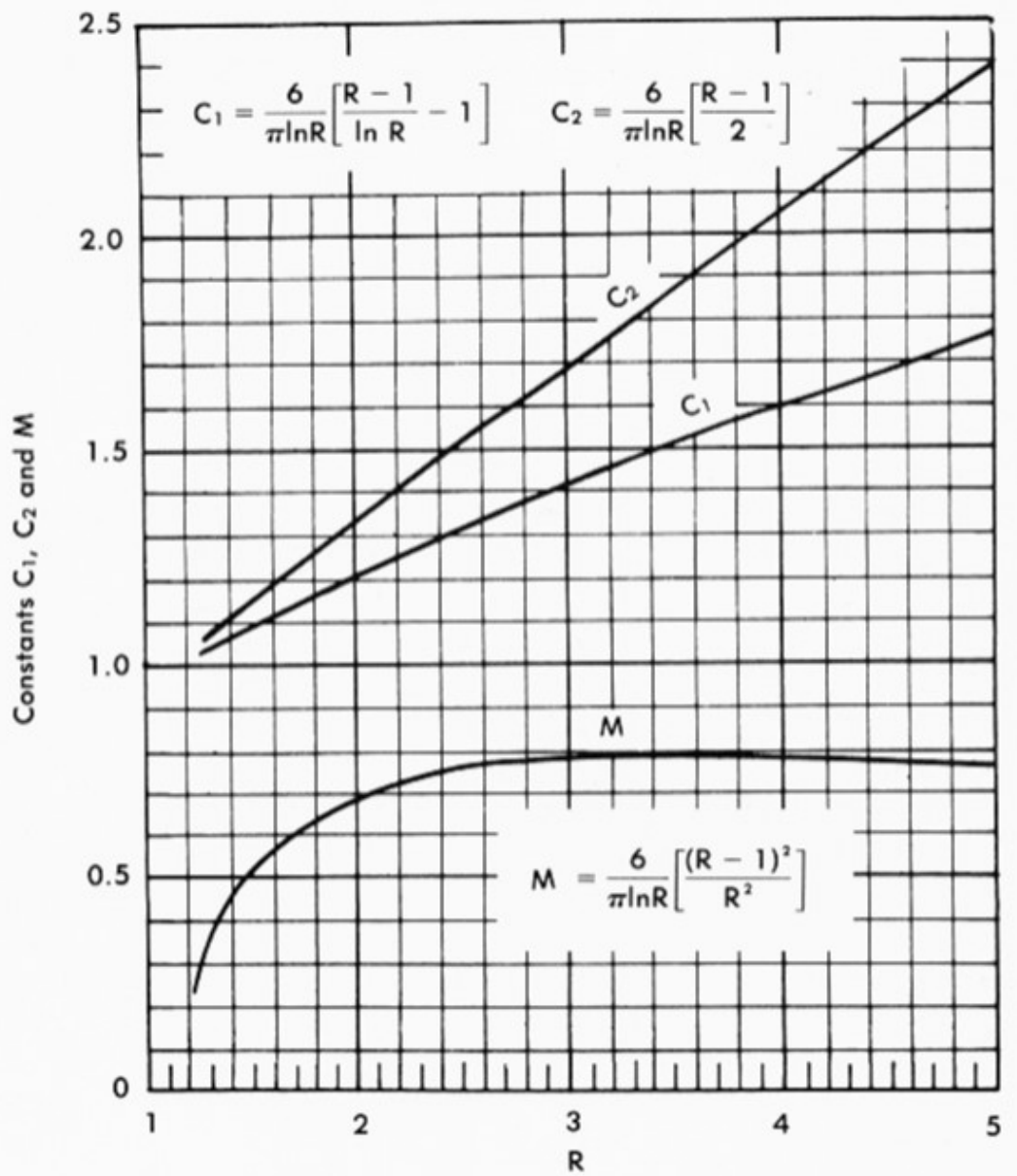
**Fig. 11-4.** Highest Stressed Regions in Belleville Washers.



**Fig. 11-5.** Comparison of  $S_{T1}$  and  $S_{T2}$  for Various Deflections,  $h/t$  Ratios and Diameter Ratios ( $R$  Values) of Belleville Washers.



**Fig. 11-6. Compressive Stress Constants for Belleville Washers.**



**Fig. 11-7. Tensile Stress Constants for Belleville Washers.**

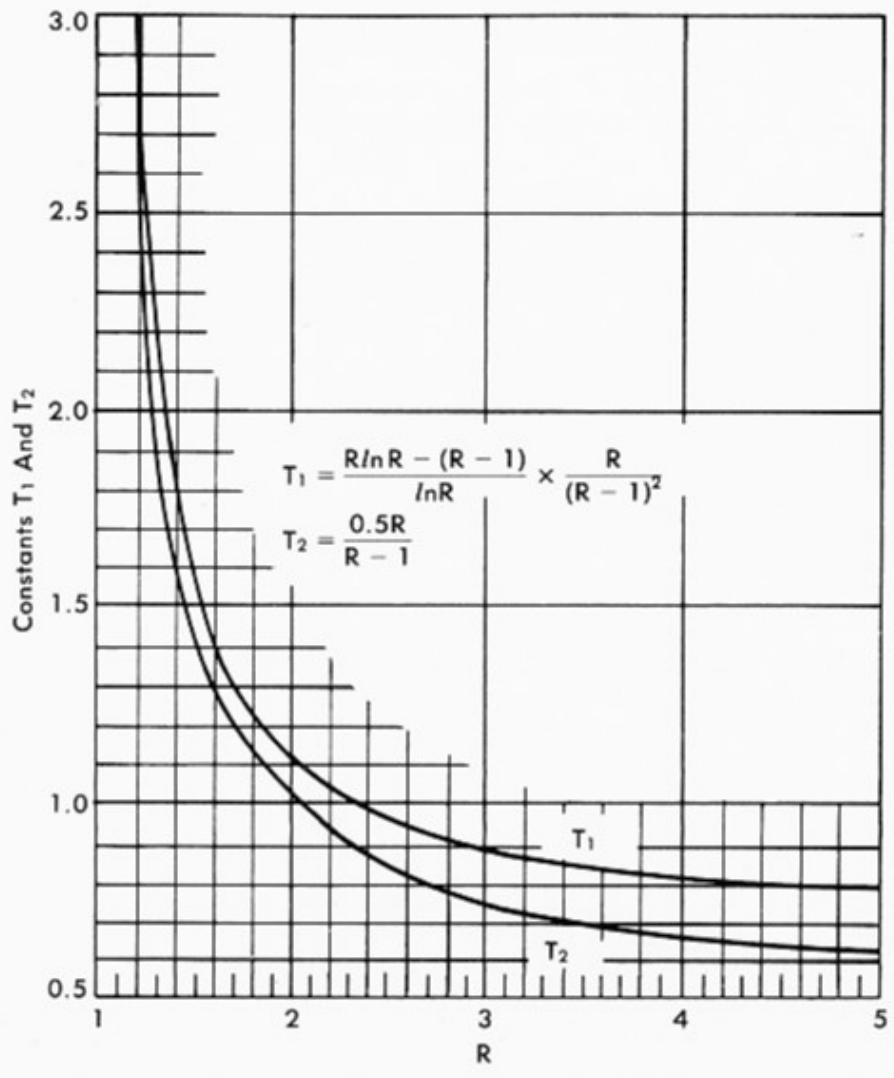
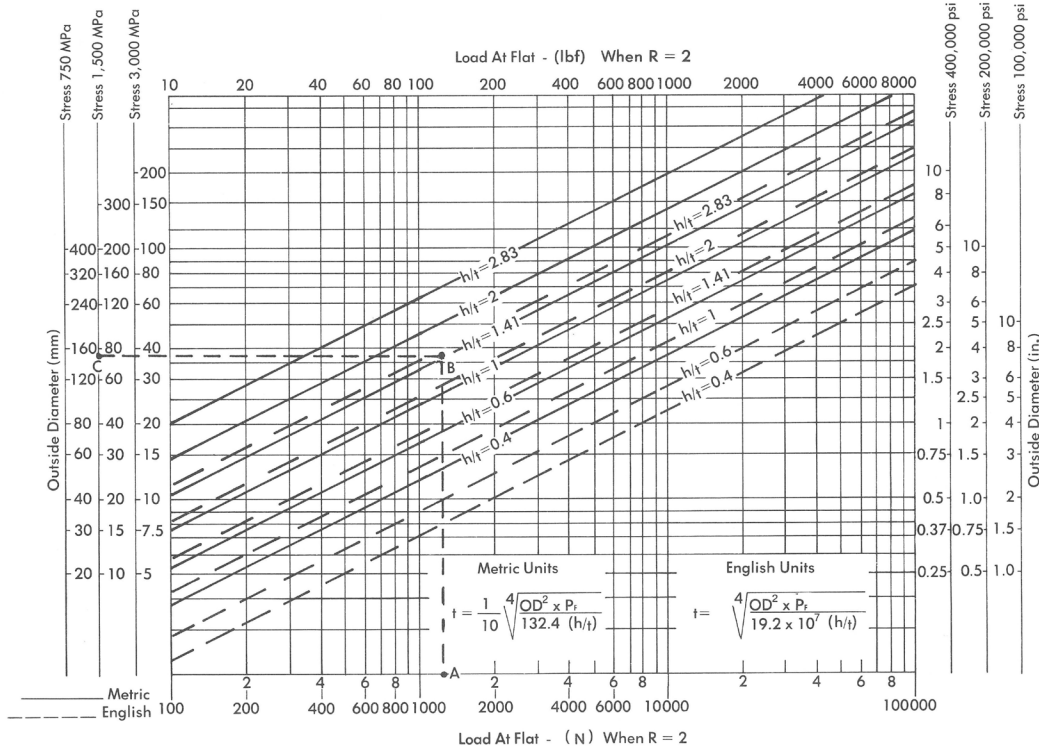


Fig. 11-8. Loads and Compressive Stresses  $S_c$  for Steel Belleville Washers with Various Outside Diameters and  $h/t$  Ratios.



Determining an optimum solution to a Belleville washer design problem is a trial-and-error process that may have to be repeated many times. A simple approach, designed to minimize the number of repetitions, is presented below. All the graphs are based on Belleville washer designs with a ratio of O.D. to I.D. of 2 ( $R = 2$ ). Designs that have  $R$  approximately equal to 2 have maximum ESC.

The first step is to select an appropriate  $h/t$  ratio based on the load, outside diameter, and stress constraints given. For example (referring to Figure 11-8), assume the desired load at flat is 1125 N and outside diameter is 76 mm. A washer with an  $h/t$  equal to 1.41 would have a maximum stress  $S_c$  of 1500 MPa. Loads at intermediate

deflection can readily be computed with the aid of Figure 11-9. Material thickness is then determined from:

$$t = \frac{1}{10} \sqrt[4]{\frac{OD^2 \times P_F}{132.4 (h/t)}} \text{ metric}$$

$$t = \sqrt[4]{\frac{OD^2 \times P_F}{19.2 \times 10^7 (h/t)}} \text{ English}$$

Before finalizing a design based on these graphs, it is best to check results using the equations, making final adjustments as required. For cyclic applications, stress levels  $S_{T1}$  and  $S_{T2}$  must be determined in order to estimate Belleville washer life.

### 8.4 Choice of Stress Level—Static

For static applications, stress at the convex inner corner  $S_c$  usually controls the spring set point. Carbon steel Belleville washers will start



to set when stress ( $S_c$ ) reaches 120% of tensile strength (Table 11-1). Set is removed in most Belleville washers, and in this case stress ( $S_c$ ) can reach 275% of tensile strength before additional set occurs. These calculated stresses are considerably higher than actual stresses due to yielding. If washers are to be plated or operated at elevated temperatures, these values must be reduced.

### 8.5 Choice of Stress Level—Cyclic

For cyclic applications, it is necessary to consider both the stress level and stress range at the concave corners  $S_{T1}$  and  $S_{T2}$ . Minimum and maximum stress must be evaluated at both  $S_{T1}$  and  $S_{T2}$  using a modified Goodman diagram. The location with the more severe conditions will control washer life. The modified Goodman diagram (Figure 11-10) illustrates fatigue strength for various thicknesses of carbon and alloy steel washers at HRC 47 to 49. Shot-peening increases fatigue strength while burrs, edge cracks, and surface imperfections reduce it.

### 8.6 Stacks of Belleville Washers

To increase deflection or loads, Belleville washers can be used in series, parallel or a combination of series and parallel (Figure 11- 11). Deflection for a series stack of identical Belleville washers is equal to the number of washers multiplied by the deflection of one, while the load is equal to the load of one washer. When Belleville washers with an h/t ratio greater than 1.3 are used in a stack, the load deflection curve will be erratic because some washers will snap through the flat position. To avoid this problem, the h/t ratio for each washer in a series stack should not exceed 1.3.

Load on a parallel stack of identical Belleville washers is equal to the load of one multiplied by the number of washers, while deflection is equal to the deflection of one washer.

The load deflection curve for both series and parallel stacks has hysteresis due to friction. Hysteresis (Figure 11-12) is greater for parallel than series stacks and can be minimized by lubrication. The energy absorbed by this hysteresis helps to dampen vibrations. By careful selection, stacks can be designed with increasing, approximately linear, or decreasing rates. Stacked Belleville washers must be guided either over a pin or in a tube. Hardness of the guides should be at least HRC 50 to minimize wear. Clearance between washer holes and pin or tube should be about 1.5% of the relevant diameter.

Fig. 11-9. Load Deflection Characteristics for Belleville Washers.

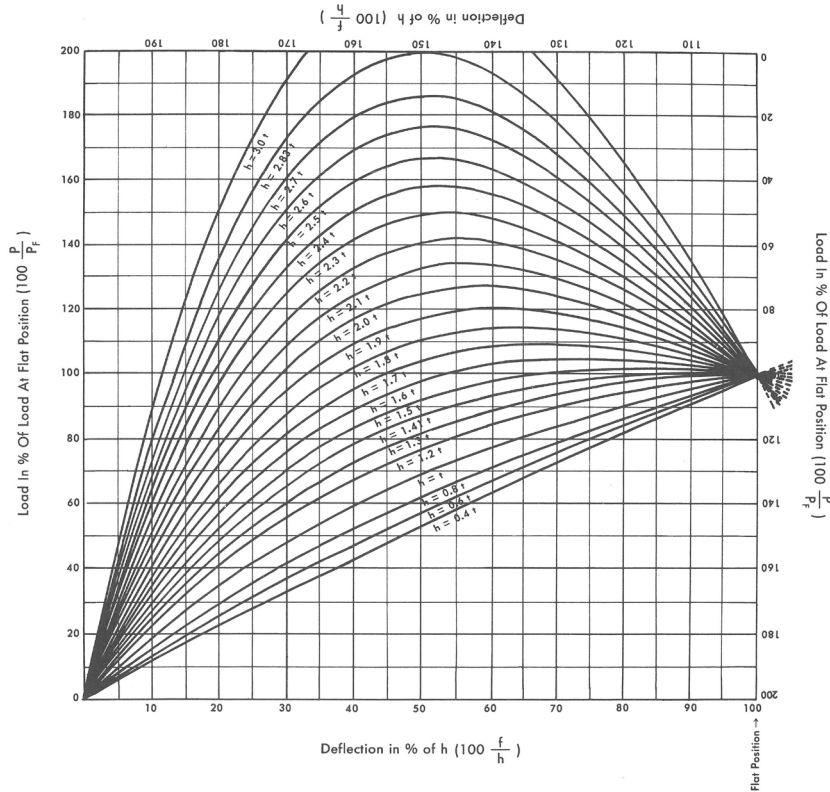
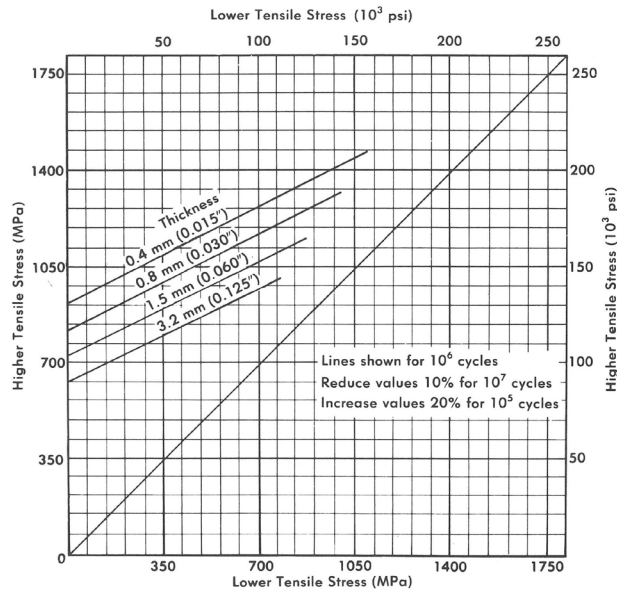


Fig. 11-10. Modified Goodman Diagram for Fatigue Strength of Belleville Washers. Carbon and Alloy Steel at HRC 47-49 with Set Removed but Not Shot-Peened.



**Table 11-1. Maximum Recommended Stress Levels  $S_c$  for Belleville Washers in Static Applications.**

Material	Percent of Tensile Strength	
	Set Not Removed	Set Removed
Carbon or Alloy Steel	120	275
Nonferrous and Austenitic Stainless Steel	95	160

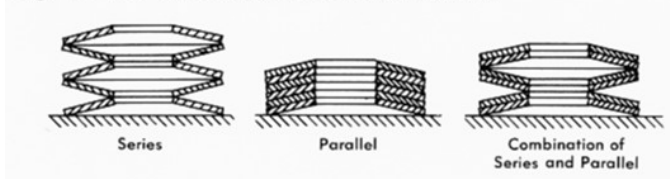
### 8.7 Tolerances

To ensure proper clearance, it is good practice to specify outside diameter with a minus-only tolerance and inside diameter with a plus-only tolerance. Recommended tolerances are shown in Table 11-2. Load tolerances should be specified at a test height. For Belleville washers with  $h/t < 0.25$ , recommended load tolerances are  $\pm 15\%$ . For washers with  $h/t > 0.25$ , use  $\pm 10\%$ . The recommended tolerance for washers made of nonferrous materials generally is  $\pm 15\%$ . Closer diameter and load tolerances are available.

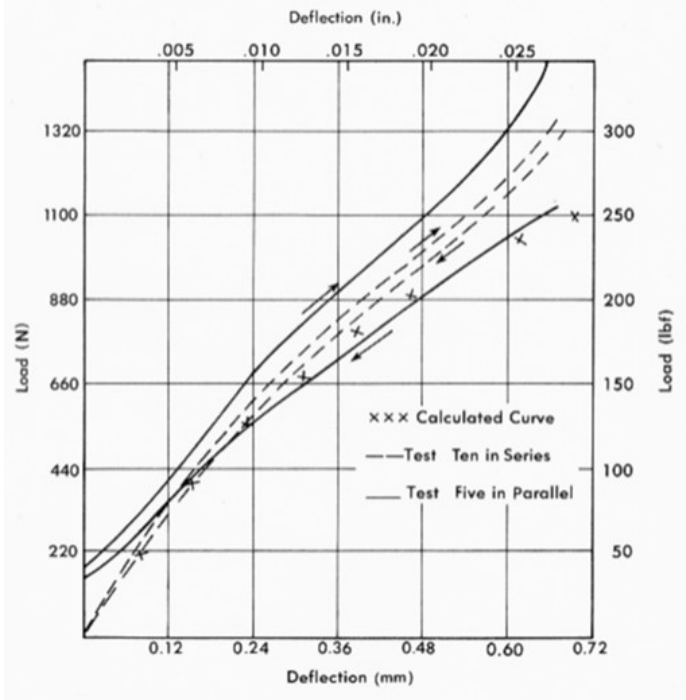
### 8.8 How to Specify

The Belleville spring washer specification checklist is attached to aid the spring designer in specifying Belleville spring washers. For washers with critical load requirements, it is recommended that a test fixture be developed.

**Fig. 11-11. Stacks of Belleville Washers.**



**Fig. 11-12. Hysteresis in Stacked Belleville Washers.**



**Table 11-2. Belleville Washer Diameter Tolerances.**

Diameter, mm (in.)	O.D. mm (in.)	I.D. mm (in.)
	+0.00	-0.00
Up to 5 (0.197)	-0.20 (-0.008)	+0.20 (+0.008)
5-10 (0.197-0.394)	-0.25 (-0.010)	+0.25 (+0.010)
10-25 (0.394-0.984)	-0.30 (-0.012)	+0.30 (+0.012)
25-50 (0.984-1.969)	-0.40 (-0.016)	+0.40 (+0.016)
50-100 (1.969-3.937)	-0.50 (-0.020)	+0.50 (+0.020)

Based on R = 2, increased tolerances are required for lower R ratios.

### 8.9 Belleville Spring Washer Design Example

In a clutch, a minimum pressure of 900 N (202 lbf) is required. This pressure must be held as nearly constant as possible while the clutch facing wears down 0.80 mm (0.031"). The washer O.D. is 76 mm (2.99"). Material selected for this application is carbon steel HRC 47-50.

1. Base the load on a value 10% above minimum load, or  $900 + 10\% = 990$  N. Assume O.D./I.D. = 2. From Figure 11-9, select a load deflection curve that gives approximately constant load between 50% and 100% of deflection to flat.



Choose the  $h/t = 1.41$  curve.

2. From Figure 11-9, the percent load at 50% deflection to flat is 88%.

3. Flat load is  $P_F = 990/0.88 = 1125$  N

4. Using Figure 11-8 (follow line A-B from 1125 N to  $h/t = 1.41$ , and line B-C to approximately 76 mm O.D.), estimated stress is 1500 MPa.

5. From Table 11-1, maximum static stress without set removed is 120% of tensile strength. From Table 191, tensile strength at HRC 48 will be approximately 1650 MPa. Yield point without residual stress will be  $1650 \times 1.20 = 1980$  MPa. Therefore, stress 1500 MPa is less than maximum stress of 1980 MPa.

6. Stock thickness is as follows:

$$t = \frac{1}{10} \sqrt{\frac{OD^2 \times P_F}{132.4 \frac{h}{t}}}$$

$$t = 0.1 \sqrt{\frac{76^2 \times 1125}{(132.4)(1.41)}} = 1.37 \text{ mm}$$

7.  $h = 1.41 t = 1.41 \times 1.37 = 1.93$  mm

$H = h + t = 1.93 + 1.37 = 3.30$  mm

8. Referring to Figure 11-9, the load of 990 N will be reached at  $f_1 = 50\%$  of maximum available deflection.  $f_1 = 0.50 \times 1.93 = 0.97$  mm deflection, or the load of 990 N will be reached at  $H_1 = H - f_1 = 3.30 - 0.97 = 2.33$  mm height at load. To allow for wear, the spring should be preloaded at  $H_2 = H_1 - \Delta f$  (wear)  $= 2.33 - 0.80 = 1.53$  mm height. This preload corresponds to a deflection  $f_2 = H - H_2 = 3.30 - 1.53 = 1.77$  mm. Then  $f_2/h = 1.77/1.93 = 0.92$  or 92%.

9. Because 92% of  $h$  exceeds the recommended 85% (the load-deflection curve is not reliable beyond 85% deflection when a washer is compressed between flat surfaces), increase the

deflection range to 40% to 85%. From Figure 11-9, the percent load at 40% deflection is 78.5% and  $P_F = 990 \div 0.785 = 1261$  N. Repeat previous procedures 4, 5, 6, 7, and 8, and find that  $f_2 \div h \times 100 = 81\%$  of  $h$ .

## 8.10 Final Design Specifications

- Material: AISI 1074, 1075
- O.D.:  $76 + 0.00, -0.05$  mm ( $2.99 + 0.00, -0.020$ " )
- I.D.:  $38 + 0.040, -0.00$  mm ( $1.50 + 0.016, -0.00$ " )
- Thickness  $t$ : 1.40 mm (0.055" ) Reference
- Height  $h$ : 1.97 mm (0.078" ) Reference
- Load: 990 N (223 lbf)  $\pm 10\%$  at  $h_1 = 1.18$  mm (0.046" )
- Compressive  $S_c$ : 1276 MPa (185,000 psi) at  $f_2$  85% of  $h$
- Tensile Stress  $S_{T1}$ : -203 MPa (-29,500 psi) at  $f_2$  85% of  $h$
- Tensile Stress  $S_{T2}$ : +710 MPa (103,000 psi) at  $f_2$  85% of  $h$

**BELLEVILLE SPRING WASHER SPECIFICATION CHECKLIST**

(Fill in required data only.)

Material \_\_\_\_\_

**Working Conditions:**

To work in \_\_\_\_\_mm(in.) diameter hole

To work over \_\_\_\_\_mm(in.) diameter pin

Load \_\_\_\_\_ N (lbf),  $\pm$  \_\_\_\_\_ N (lbf)

Test height \_\_\_\_\_mm(in.)

Relaxation \_\_\_\_\_%

Required life \_\_\_\_\_cycles.

Required reliability (see Section 4) \_\_\_\_\_

**Special Information:**

Maximum operating temperature \_\_\_\_\_ °C(°F)

Operating environment \_\_\_\_\_

To be used in a stack (type) \_\_\_\_\_

**Reference Data:**

Thickness \_\_\_\_\_mm(in.)

Outside Diameter \_\_\_\_\_mm(in.)

Inside Diameter \_\_\_\_\_mm(in.)

Free height \_\_\_\_\_mm(in.)

h/t \_\_\_\_\_

## 9 FLAT SPRINGS

### 9.1 Introduction

For purposes of classification, the term “flat spring” refers to all springs made from sheet, strip, or plates that are not of another category such as washers or power springs. Flat springs may contain bends and complicated forms. Thus, the term “flat” is not an accurate description of the spring itself. Flat springs are produced in an unlimited variety of shapes from a wide assortment of materials.

Flat springs can perform functions beyond normal spring functions. A flat spring may be required to conduct electricity, act as a latch, or hold a part in position. Because flat springs are custom designed to perform several functions, standard designs are not available, and tooling is often a major cost consideration.

Design techniques for flat springs vary in complexity. Simple methods, discussed in this handbook, use familiar cantilever and simple beam equations to determine stress and deflection. Recognizing that the neutral axis of a curved member is not at the centroid, more accurate results are obtained with equations based on curved beam theory. If the amount of elastic deflection is of interest, the Castigliano method is helpful. For a complete description of these methods, refer to “Design of Curved Members for Machines” by Blake (Reference 6). More recently, methods based on finite element analysis have been successfully employed for springs subject to relatively small deflections.

### 9.2 Design Considerations

Most flat springs are loaded in bending. The stress for relatively wide strip is highest at the

surface between the two edges. For cyclic applications, a smooth surface is required. Sharp edges or burrs should be eliminated by abrasive finishing after blanking or avoided by use of edge-finished strip. Sharp bends are not only difficult to form but act as stress concentrators and should be avoided. Ductility of strip is greatest in a direction parallel to the rolling direction; therefore, the bend axis should be perpendicular to the rolling direction. Pre-strengthened material is often more cost-effective than material that is hardened after forming because parts distort during hardening, sometimes requiring fixture tempering.

A detailed drawing with notes on the functional requirements of a part is the best way to specify a flat spring. Although loads specified at a test height are easier to control than loads specified at a deflection, many designers specify loads at a deflection.

### 9.3 Design Equations

Design equations for flat springs are based on Bernoulli-Euler Beam Theory for the bending of beams subjected to small deflections. The maximum stress is:

$$(12-1) \quad S = \frac{Mc}{I}, \text{ where } c \text{ is one - half the thickness.}$$

For beam springs with a relatively low width-to-thickness ratio, the maximum stress and deflection, as given by the formulas below, are reasonably accurate. As the width-to-thickness ratio increases, however, the lateral deformation that would normally accompany the longitudinal fiber stresses becomes restricted, effectively increasing the flexural rigidity of the spring. This increased stiffness is accounted for by replacing

the elastic modulus (E) by the following in the beam deflection equations (12-4, 12-6, and 12-7):

(12-2)

$$\frac{E}{1 - \mu^2}$$

When the width-to-thickness ratio is greater than 10 and the length-to-width ratio is less than 5, this factor should be applied unless the load is distributed along the width of the spring. For example, with spring steel having  $\mu = 0.30$  and a load concentrated at the center, the modulus should be increased by about 10%.

## 9.4 Cantilever Springs

The most basic type of flat spring is the rectangular cantilever spring. It is usually mounted as shown in Figure 12-1. Maximum bending stress occurs at the clamping point and is calculated from the following:

(12-3)

$$S = \frac{6PL}{bt^2}$$

Where the load is given by:

(12-4)

$$P = \frac{fEbt^3}{4L^3}$$

These equations are only satisfactory when the ratio of deflection to cantilever length ( $f/L$ ) is less than 0.3. For large deflections, a method is shown in Figure 12-2.

In cantilever springs with a trapezoidal section (Figure 12-3), stress is uniform throughout. In general, benefits gained by using a trapezoidal section are offset by the increased production cost of this special shape. Stress for trapezoidal cantilever springs is determined from:

(12-5)

$$S = \frac{6PL}{b_0t^2}$$

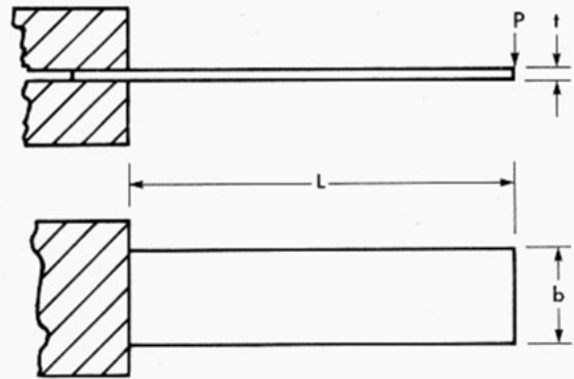
While the load is given by:

(12-6)

$$P = \frac{fEb_0t^3}{4L^3K}$$

K is a constant based on the ratio of  $b/b_0$  (Figure 12-4). These equations are also only valid for small deflections where the ratio of deflection to cantilever length is less than 0.3.

Fig. 12-1. Rectangular Cantilever Spring.



## 9.5 Simple Beam or Elliptical Springs

Simple beams are usually rectangular in shape and formed into an arc as illustrated in Figure 12-5. Load is usually applied at the center of the arc. Such a spring is not subject to higher stresses at the clamping point as is a cantilever spring. If the spring is clamped or pierced to provide a precise location, stress increases will be encountered at the clamp or hole. When the ends are free to move laterally, deflection in the direction of loading is related to load by:

(12-7)

$$P = \frac{4fEbt^3}{L^3}$$

Stress is given by:

(12-8)

$$S = \frac{1.5PL}{bt^2}$$

These equations are only satisfactory for small deflections where the ratio of deflection to beam length  $f/L$  is less than 0.15.

### 9.6 Choice of Operating Stress

Maximum recommended operating stress levels are presented in Table 12-1 for static applications and Table 12-2 for cyclic applications. Allowances must be made for stress concentrations due to clamping, burrs, and sharp bends for particular applications. A shift in position of the neutral axis causes a stress concentration at the inner surface of:

$$K_{ID} = \frac{3C^2 - C - .8}{3C(C - 1)}$$

and at the outer surface of:

$$K_{OD} = \frac{3C^2 - C + .8}{3C(C + 1)}$$

C is the index as shown in Figure 12-6.

Holes, corners, notches, and abrupt changes in cross-section cause stress concentrations. The theoretical stress correction factor for the case of a transverse hole in wide strip is illustrated in Figure 12-7, where the nominal stress is calculated for the net section.

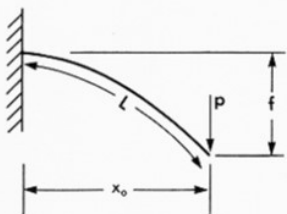
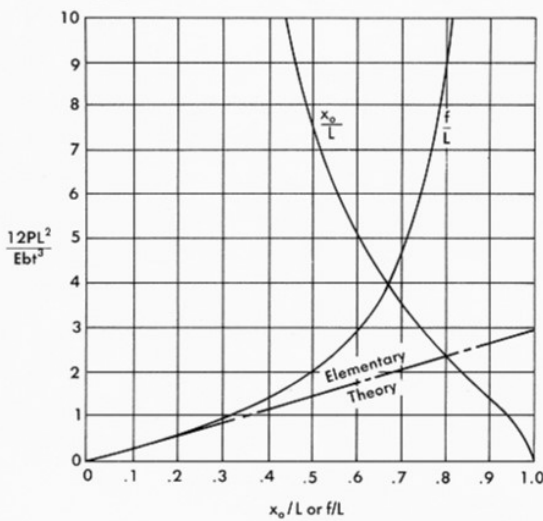
### 9.7 Tolerances

Tolerances for the blanked dimensions of flat springs are the same as tolerances for any metal stamping. Forming tolerances depend on the material and part configuration. Angular tolerances, due to forming, should be greater than  $\pm 2^\circ$  whenever possible. Thickness tolerances are equal to strip thickness tolerances presented previously (Table 3-12). Load tolerances also depend on part configuration but generally should not be less than  $\pm 10\%$ . Associated Spring engineers should be consulted for specific tolerance information.

### 9.8 How to Specify

Because of the infinite variety of shapes and functions of flat springs, an engineering drawing is the best way to specify a part.

Fig. 12-2. Calculating Large Deflection in Cantilever Beams. (After Bisshopp and Drucker, Reference 7, page 102)



To use Fig. 12-2:  
 Calculate  $\frac{12PL^2}{Ebt^3}$   
 Read values for  $\frac{f}{L}$  and  $\frac{x_0}{L}$  from horizontal axis  
 Deflection  $f = \left(\frac{f}{L}\right)L$   
 Stress  $s = \left(\frac{x_0}{L}\right)\frac{6PL}{bt^2}$

Fig. 12-3. Trapezoidal Cantilever Spring.

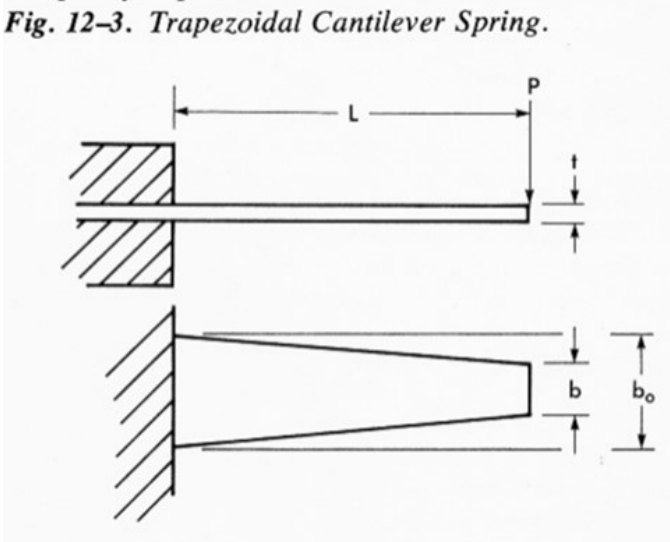
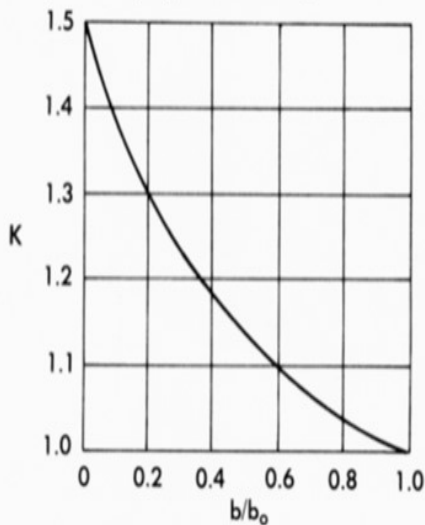


Fig. 12-4. Correction Factor for Trapezoidal Beam Load Equation (Equation 12-6).



### 9.9 Flat Spring Design Example

Design a cantilever spring to exert a force of  $5.0 \pm 0.5$  N ( $1.1 \pm 0.1$  lbf) at a distance of 100 mm (3.937") from a clamp. The spring will be deflected an additional 5 mm to open an electrical circuit. A 1 million cycle life with a 90% survival rate is desired. Maximum width is 10 mm. The spring operates in an ambient environment; therefore, AISI 1075 was selected.

1. Assume that the load in the fully deflected position is 60% greater than the load in the static position and that width is 8 mm. From the rate equation:

$$\frac{P}{f} = \frac{Ebt^3}{4L^3} = \frac{3}{5} = \frac{(207,000)(8)t^3}{(4)(100)^3}$$

Thickness is 1.13 mm (select  $t = 1.10$ ).

2. Calculate stress:

$$S = \frac{6PL}{bt^2} = \frac{(6)(8)(100)}{(8)(1.1)^2} = 496 \text{ MPa}$$

TS for strip with high strength (Figures 3-4 and 3-6, page 21) is 1480 MPa (215,000 psi). Maximum stress divided by tensile strength equals 34%. Table 12-1 indicates that this is a safe operating stress. Maximum deflection is 8 divided by 0.6 which equals 13.3 mm. This is a small deflection since ratio  $f/L$  is less than 0.3.

### 9.10 Final Design Specifications

- Length L: 100 mm (3.937")
- Width b: 8 mm (0.315")
- Thickness t: 1.1 mm (0.043")
- Load  $P_1$ :  $5 \pm 0.5$  N ( $1.12 \pm 0.11$  lbf)
- Load  $P_2$ : 8 N (1.80 lbf) Reference

Table 12-1. Maximum Design Stresses for Cantilever and Simple Beam Springs in Static Applications.

Percent of Tensile Strength			
Ferrous Material		Nonferrous Material	
No Residual Stress	Maximum Residual Stress	No Residual Stress	Maximum Residual Stress
80	100	75	80

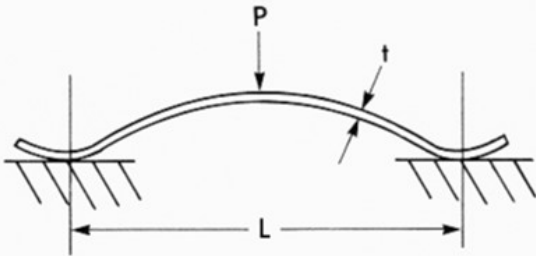


**Table 12-2. Maximum Design Stresses for Carbon Steel Cantilever and Simple Beam Springs in Cyclic Applications.**

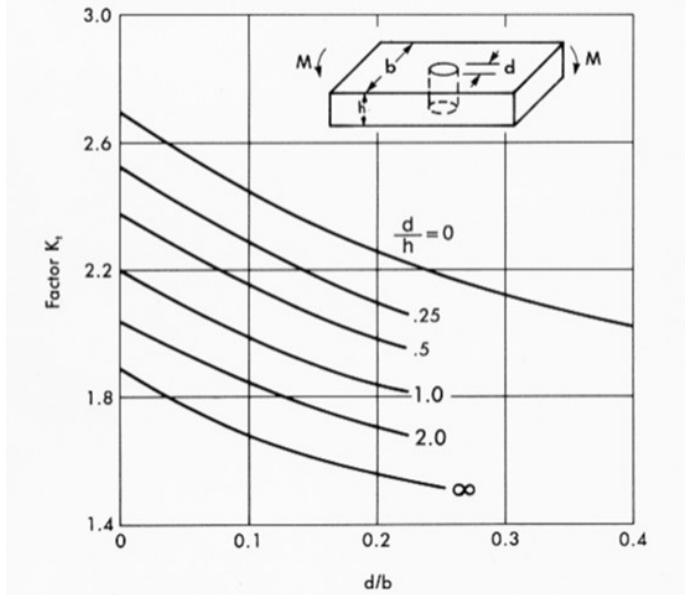
Number of Cycles	Percent of Tensile Strength	
	Not Shot-Peened	Shot-Peened*
10 <sup>5</sup>	53	62
10 <sup>6</sup>	50	60
10 <sup>7</sup>	48	58

\*Shot peening is not recommended for thin materials and complex shapes. This information is based on an ambient environment. Stress ratio = 0.

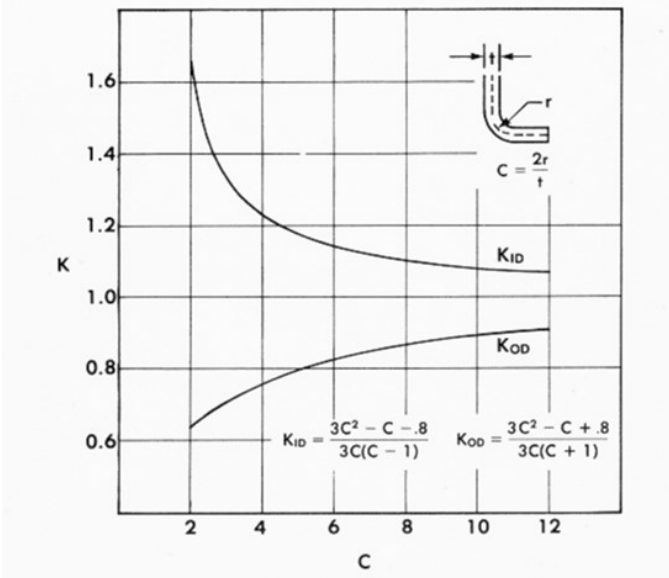
**Fig. 12-5. Simple Beam Spring.**



**Fig. 12-7. Stress Concentration Factor ( $K_t$ ) for Bending at Transverse Hole.** (After Peterson, Reference 10, page 102.)



**Fig. 12-6. Stress Correction due to Shift in Neutral Axis at Sharp Bends.**



## 10 SPECIAL SPRING WASHERS

### 10.1 Introduction

Special spring washers exert a thrust load and absorb vibration, reduce end play, or apply pressure. The state of stress is primarily bending, and most of the general design considerations for flat springs (Section 11) apply. Spring washers are used in seals, bearings, motors, and other rotating mechanisms, and because of the trend toward miniaturization and compactness, demand for them is increasing.

The Associated Spring RAYMOND SPEC product line contains many precision-engineered wave, curved, and finger spring washers. These washers are made to close tolerances and are available for immediate delivery. Selecting SPEC washer designs saves design time, avoids tooling costs, and is generally more cost-effective than specifying custom-designed parts.

### 10.2 Curved Washers

Curved washers exert a relatively light thrust load and are often used to absorb axial end play. Designers must provide space for diametral expansion in a direction perpendicular to the A dimension (Figure 13-1). Bearing surfaces should be hard to prevent washer corners from scraping or digging in. The spring rate is approximately linear up to 80% of available deflection. Beyond 80%, the rate increases and is considerably higher than calculated.

Design equations for spring washers are similar to those for simple beams, discussed in Section 11, except for an empirical correction factor K. The equation for load is:

(13-1)

$$P = \frac{4fEt^3}{(OD)^2K}$$

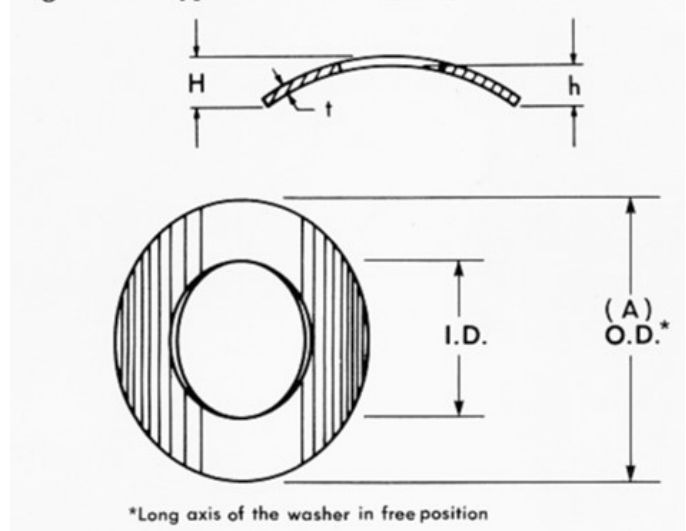
O.D. is outside diameter in the flat position and the equation for stress is:

(13-2)

$$S = \frac{1.5P}{t^2}K$$

Correction factor K is shown in Figure 13-2. These equations are approximate and yield satisfactory solutions only for deflections up to 80% of h where f is less than one-third of O.D. Associated Spring engineers should be consulted when clearances are critical or more exact designs are required.

Fig. 13-1. Typical Curved Spring Washer.



### 10.3 Wave Washers

Wave spring washers, Figure 13-3, are especially useful to apply moderate thrust loads when radial space is limited. The rate is linear between 20% and 80% of available deflection. During forming, the washer is often stretched at the crest and trough of the waves. Washers that are round in the free position go out-of-round when deflected. Generally, a ratio of  $D/b = 8$  is a good

balance between flexibility and load-carrying ability. When the ratio of  $D/b$  is substantially lower than 8, a Belleville washer is preferred.

The number of waves  $N_a$  can be equal to 3 or more and is usually selected on the basis of desired spring rate, since spring rate is proportional to the number of waves raised to the fourth power, as:

$$(13-3) \quad k = P/f = \frac{Ebt^3N_a^4D_o}{2.40D^3D_t}$$

This formula is based on the equations for a simple beam with correction factors based on experience to improve accuracy. Stress is given by:

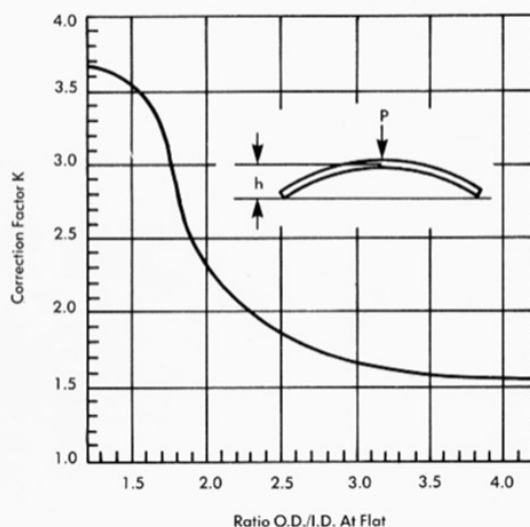
$$(13-4) \quad S = \frac{3\pi PD}{4bt^2N_a^2}$$

The outside diameter of the washer changes upon deflection, and at flat is given by:

$$(13-5) \quad D_o' = \sqrt{D_o^2 + 0.458h^2N_a^2}$$

$D_o$  is the outside diameter in the free position. The above equations for load, stress, and diametral change are not exact solutions, but do provide useful engineering estimates for design purposes.

Fig. 13-2. Empirical Stress Correction Factor  $K$  for Curved Spring Washers.



## 10.4 Finger Washers

Finger washers, Figure 13-4, combine the flexibility of curved washers and the distributed loading points of wave washers. Load, deflection, and stress are approximated by assuming that the fingers are cantilever springs; then, samples are made and tested to prove the design. Finger washers are used in static applications such as applying an axial load to ball bearing races to reduce vibration and noise.

## 10.5 Choice of Operating Stress—Static

Operating stresses recommended for special spring washers are similar to stress levels recommended for flat washers, which are similar to stress levels recommended for flat springs, and are shown in Table 13-1 as a percent of tensile strength. Finger washers are generally produced in the stress-relieved condition. If favorable residual stresses are required, consult Associated Spring.

## 10.6 Choice of Operating Stress—Cyclic

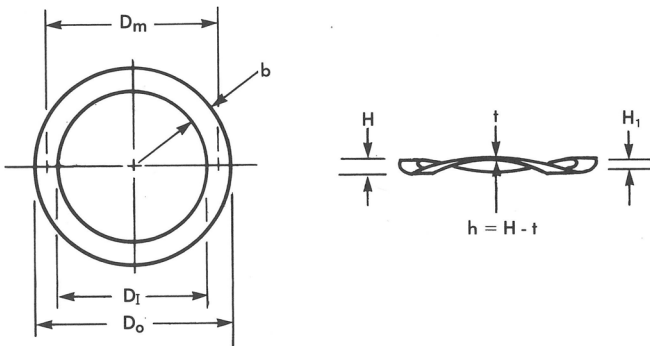
Maximum recommended operating stresses for cyclic conditions are shown in Table 13-2 for curved and wave washers. Finger washers are not recommended for cyclic applications.

## 10.7 Tolerances

Dimensional tolerances are similar to those on flat springs. Load tolerances depend primarily on strip thickness tolerances and are listed in Table 13-3. All load tolerances should be specified at a test height and only those dimensions critical to spring function should have tolerances. Special tolerances are available for demanding

applications.

**Fig. 13-3. Typical Wave Spring Washer.**



**Table 13-1. Maximum Recommended Operating Stress Levels for Special Spring Washers in Static Applications.**

Material	Percent of Tensile Strength	
	Stress-Relieved	With Favorable Residual Stresses
Steels, Alloy Steels	80	100
Nonferrous Alloys and Austenitic Steel	—	80

Finger washers are not generally supplied with favorable residual stresses.

### 10.8 How to Specify

The special spring washer specification checklist is provided as a guide to all critical aspects of special spring washers.

### 10.9 Special Spring Washer Design Example

A wave washer is needed to go into an 80 mm (3.15") bore and over a 60 mm (2.362") shaft, to support a load of approximately 500 to 550 N (112 to 124 lb) with 1.8 mm (0.071") deflection. The application requires a steady load and is therefore a static application. The washer will operate in an ambient environment. AISI 1075 is the preferred material.

Since deflection is comparatively large for a spring of this type, select the most flexible design—three-wave configuration.

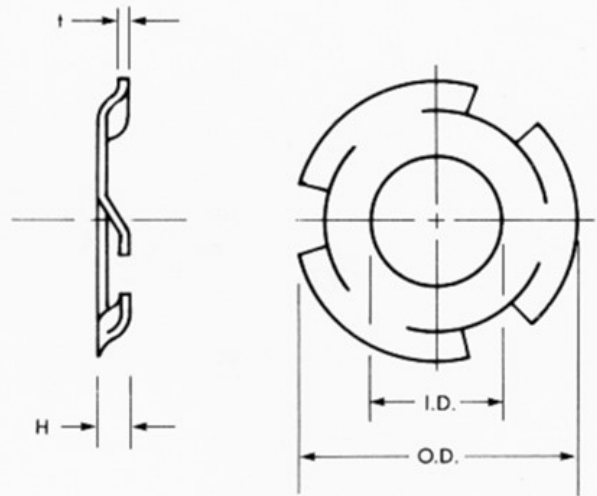
Assume a 75 mm (2.953") outside diameter and a 64 mm (2.320") inside diameter to fit the given conditions. This would make the mean diameter (D) 69.5 mm (2.736").

Substituting these values in the load-deflection equation, solve for thickness:

$$t = \sqrt[3]{\frac{(2.40)(525)(69.5)^3(64)}{(207,000)(1.8)(5.5)(3)^4(75)}} = 1.30 \text{ mm}$$

$$P_1 = \frac{Efbt^3N_a^4 \left(\frac{D_o}{D_1}\right)}{2.4 D^3} = 530 \text{ N}$$

**Fig. 13-4. Typical Finger Spring Washer.**



**Table 13-2. Maximum Recommended Operating Stress Levels for Steel Curved and Wave Washers in Cyclic Applications.**

Life (Cycles)	Percent of Tensile Strength
	Maximum Stress
10 <sup>4</sup>	80
10 <sup>5</sup>	53
10 <sup>6</sup>	50

This information is based on the following conditions: ambient environment, free from sharp bends, burrs, and other stress concentrations. AISI 1075



Set the maximum stress at solid at 80% of tensile strength. Steel with a hardness of HRC 49 has a tensile strength of 1725 MPa (250,000 psi) (Table 13-1 and Figure 3-6). 80% of 1725 MPa is 1380 MPa. Solve for deflection at that stress using the equation:

$$f_s = \frac{9.6 D^2 S D_I}{3\pi E t N^2 D_O} = \frac{(9.6)(69.5)^2(1380)(64)}{(3)\pi(207,000)(1.3)(3)^2(75)}$$

$$f_s = 2.39 \text{ mm}$$

Deflection to load of 1.8 mm is 75% of deflection to solid, which is satisfactory. Diameter in the deflected position:

$$D_{O'} = \sqrt{D_o^2 + 0.458 h^2 N^2} = \sqrt{(75)^2 + 0.458(1.80)^2(3)^2}$$

$$D_{O'} = 75.1 \text{ mm}$$

There is adequate clearance.

**Table 13-3. Load Tolerances for Special Spring Washers.**

Washer	Stock Thickness mm (in.)	± Percent Load
Curved	0.1-1.0 (0.004-0.039)	15
Wave	0.1-0.25 (0.004-0.010)	33
	0.25-0.30 (0.010-0.012)	25
	0.3-0.5 (0.012-0.020)	20
	0.5-1.0 (0.020-0.039)	15
	1.0-2.0 (0.039-0.079)	12
Finger	Consult Associated Spring engineers.	

### 10.10 Final Design Specifications

- Material: AISI 1075
- O.D.: 75 ± 0.2 mm (2.953 ± 0.008")
- I.D.: 64 ± 0.2 mm (2.520 ± 0.008")
- Thickness t: 1.30 mm (0.055 ± 0.002")
- H: 3.69 mm (0.145") Reference
- Load P<sub>1</sub>: 530 N ± 12% (119 lbf ± 12%)
- H<sub>1</sub>: 1.89 mm (0.074")

#### SPECIAL SPRING WASHER SPECIFICATION CHECKLIST

(Fill in required data only.)

Type of Washer:

Curved , wave , finger , designer recommendation

Material: \_\_\_\_\_

Working Conditions:

To work in \_\_\_\_\_mm(in.) diameter hole

To work over \_\_\_\_\_mm(in.) diameter pin

Load \_\_\_\_\_N(lbf) ± \_\_\_\_\_N(lbf)

Test height \_\_\_\_\_mm(in.)

Required reliability (see Section 4) \_\_\_\_\_

Describe one cycle \_\_\_\_\_

Special Information:

Maximum operating temperature \_\_\_\_\_°C(°F)

Operating environment \_\_\_\_\_

Reference Data:

Thickness \_\_\_\_\_mm(in.)

Outside diameter \_\_\_\_\_mm(in.)

Inside diameter \_\_\_\_\_mm(in.)

Free height \_\_\_\_\_mm(in.)

Other \_\_\_\_\_



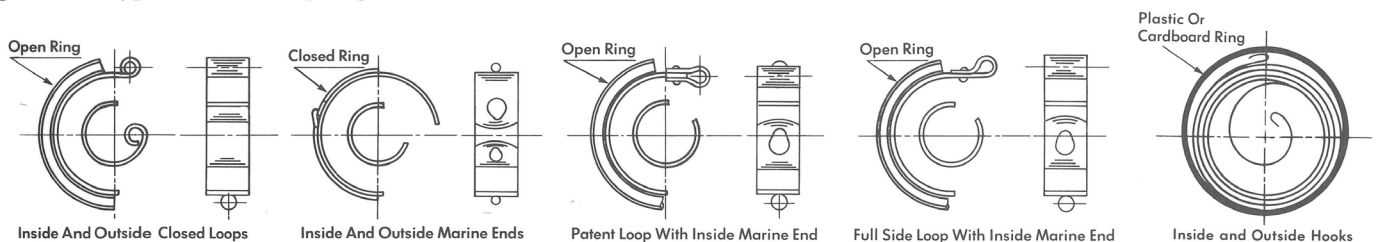
## 11 POWER SPRINGS

### 11.1 Introduction

Power springs, also known as clock, motor, or flat coil springs, consist of strip wound on an arbor and confined in a case or ring. Power springs store and release rotational energy in the form of torque, either through the central arbor or the case in which it is restrained. Power springs and pre-stressed power springs are similar in appearance. However, pre-stressed power springs have greater ESC due to residual stress distribution manufactured into the strip prior to placing the spring in its case. When a pre-stressed power spring is removed from its case, it assumes an S shape, while power springs assume a spiral shape (Figure 14-1). Although design procedures for the two types of springs are similar, manufacturing costs of pre-stressed power springs are often greater, especially in small quantities.

Because of their ability to store great amounts of rotational energy, power springs and pre-stressed power springs are used in clocks, spring motors for toys, cameras, and timers, as well as in retractor mechanisms for electrical cords and seat belts. To meet the requirements of these varied applications, power springs are produced in a wide range of sizes. They are normally supplied in retainers and have assorted end configurations to facilitate assembly. Some typical retainers and end configurations are shown in Figure 14-2.

**Fig. 14-2.** Typical Power Spring Retainers and Ends.

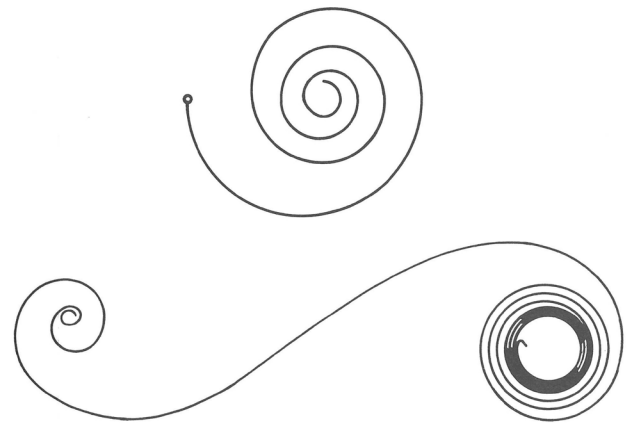


### 11.2 Design Considerations

Power springs are stressed in bending and the stress is related to torque by:  
(14-1)

$$S = \frac{6M}{bt^2}$$

**Fig. 14-1.** Typical Unrestrained Power Spring (top) and Prestressed Power Spring (bottom).



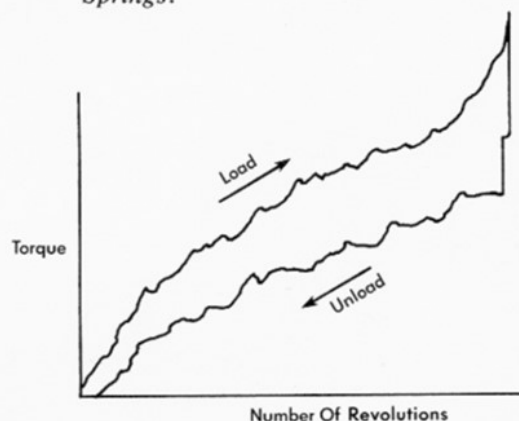
Load deflection curves for power springs are difficult to predict because the amount of active material changes as the spring is wound and because of friction between coils. Even well-lubricated springs, made by special techniques to reduce friction, exhibit irregular torque behavior and great hysteresis (Figure 14-3). On unloading to zero torque, the arbor may not always return to its original position. In practice, most springs are preloaded to assure a minimum torque at a fixed number of turns. Because of the steep slope at the start and end of the torque rotation curve, torques should be specified between 20% and 80% of total rotation.

The ratio of arbor diameter to stock thickness  $D_a/t$  is an important consideration. If the arbor is too small, the spring will set, and if  $D_a/t$  is large, it will not be possible to achieve maximum torque.  $D_a/t$  ratios of 15 to 25 are generally considered satisfactory.

A second important design consideration is case size in relation to the amount of strip. Laboratory tests have shown that for maximum ESC, a spring should occupy 40% to 50% of the available space between arbor and case. If more strip is used, the number of turns available for rotation decreases and if less strip is used, the same spring could be put into a smaller case.

The third design consideration is that rotation to solid for most power springs should be fewer than 25 turns, and the length-to-thickness ratio should be less than 15,000 because of friction. Prestressed power springs can have higher length-to-thickness ratios and more revolutions to solid.

*Fig. 14-3. Typical Torque-Revolution Curve for Power Springs.*



The final design consideration concerns expected life. Power springs operate at very high stress levels and rarely have lives greater than 200,000 cycles. Lives in the 2,000 to 100,000 cycle range are considered acceptable for most applications. Using round edge material increases life; however, for long life, it is necessary to reduce stress levels. Power springs made from 0.20 to 1.2 mm (0.008" to 0.047") pre-tempered AISI 1095 steel in the HRC 50 to 52 range can be stressed at 100% of tensile strength for lives up to approximately 10,000 cycles. If maximum stress is 50% of tensile strength, the expected life is approximately 100,000 cycles. The stress ratio is zero in both cases.

### 11.3 Design Equations

To design a power spring that will deliver a given torque and number of turns, the first step is to determine its maximum torque in the fully wound position, using a normalized torque revolution curve (Figure 14-4). If, for example, a design requires a spring to deliver a minimum torque of 0.5 N · m for 10 revolutions and assuming this occurs at the 80% unwound position, maximum torque at solid would be 1.0 N · m. A 10 mm wide, 0.58 mm (0.023") thickness is required to deliver this torque (Figure 14-5). If strip width were 20 mm, the torque per mm of width would

be cut in half, and thickness is 0.43 mm. Average maximum solid stress for this thickness is 1920 MPa (278,000 psi) from Figure 14-6.

Fig. 14-4. Typical Normalized Torque-Revolution Curve for Power Springs.

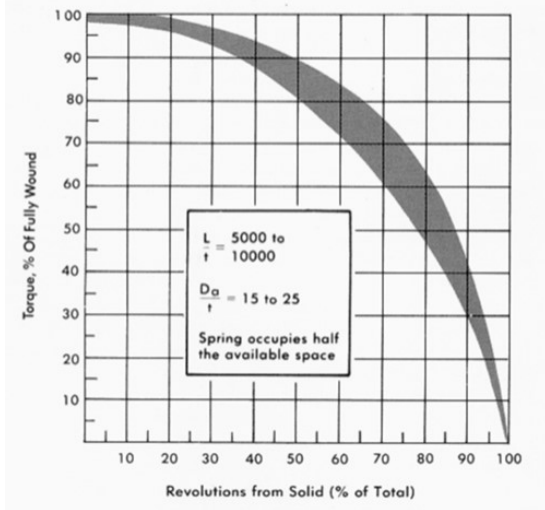
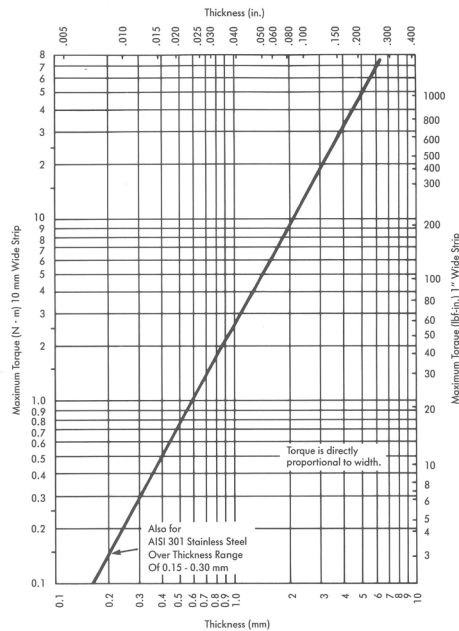


Fig. 14-5. Theoretical Torque at Solid for Carbon Steel Power Springs.



The number of turns that a power spring will deliver when it occupies one-half of the space between arbor  $D_a$  and case  $D_c$  can be estimated from the theoretical equation:

(14-2)

$$\theta = \frac{\sqrt{2(D_c^2 + D_a^2)} - (D_c + D_a)}{2.55t}$$

The actual number of turns delivered in practice is a half to one turn less than this value, due to the small diameter “free” coils near the arbor.

The spring length is:

(14-3)

$$L = \frac{D_c^2 - D_a^2}{2.55t}$$

These are optimum conditions, and the springs should not be made longer, although they may be made shorter if a reduction in the number of revolutions is acceptable.

When the ratio of space occupied by material to available space is not 50%, the following applies:

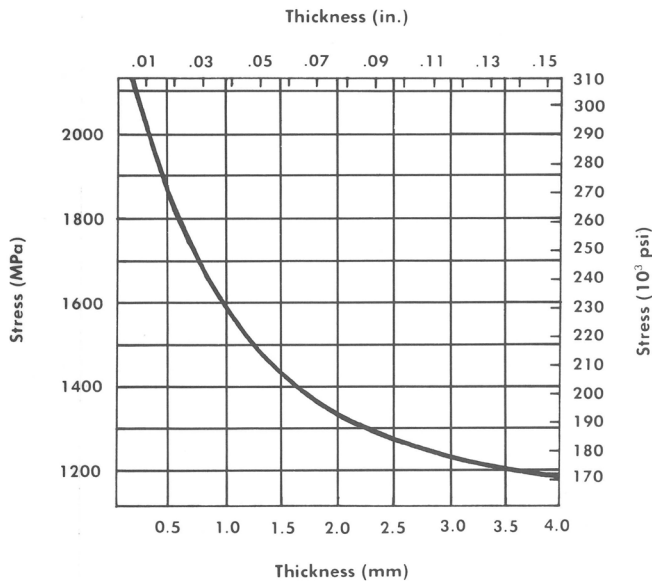
(14-4)

$$\theta = \frac{\sqrt{1.27tL + D_a^2} + \sqrt{D_c^2 - 1.27tL} - (D_c + D_a)}{2t} K$$

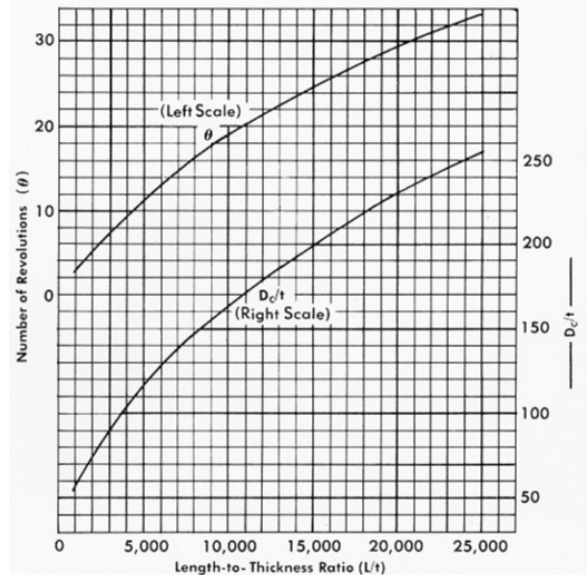
K is an empirical factor from Table 14-1. Selecting the proper levels of a variable such as arbor diameter, case diameter, number of revolutions, and strip length is a trial-and-error process. An aid to reduce the number of repetitions necessary to choose the best relationship between these variables is shown in Figure 14-7.

The design procedure for pre-stressed power springs is similar to that for power springs. Prestressed power springs have a greater ESC and can produce 25% to 55% more torque than comparable power springs. Prestressed power springs designed to deliver the same torque as regular power springs in the same space envelopes are made from thinner strip and have a greater number of turns available. A common high-volume application for pre-stressed power springs is in automotive seat belt retractors.

**Fig. 14-6.** Average Maximum Solid Stress in Carbon Steel Power Springs.



**Fig. 14-7.** Relationships Between Number of Revolutions, Case Diameter, Strip Length and Thickness for Power Springs.



Prestressed power springs are characterized by a relatively flat torque-revolution curve. The design procedure for retractor springs is to first determine the torque at solid using the generalized torque-revolution curve (Figure 14-8) for pre-stressed power springs. The strip thickness is a function of maximum torque at solid (Figure 14-9). Cross-curvature imparted to the strip minimizes inter-coil friction. The total number of turns available in a pre-stressed power spring is:

$$\theta = \frac{\sqrt{\left(\frac{D_c}{t}\right)^2 + \frac{4L}{\pi t} - \frac{D_c}{t}} - \left(4 + \frac{D_c}{t} - \sqrt{\left(\frac{D_c}{t}\right)^2 - \frac{4L}{\pi t}}\right)}{2}$$

Prestressed retractor springs occupy less than 50% of the space available. Arbor diameter is usually 35 to 55 times the material thickness and the length-to-thickness ratio is typically 20,000. Prestressed power springs have lives from 80,000 to 120,000 cycles when made from 0.20 to 0.25 mm (0.008" to 0.010") thick, high tensile AISI 301 stainless steel or carbon steel, and subjected to maximum calculated stresses in the range of 1650 to 2930 MPa (239 to 425X10<sup>3</sup> psi).

### 11.4 How to Specify

Because the performance of power springs or pre-stressed power springs is very dependent on manufacturing technique, only the arbor, case, end configurations, and functional properties should be specified and given tolerances. Material thickness and length should be reference dimensions. Torque is usually specified as either a minimum or maximum value at a given number of turns from solid. Torque can also be specified at a nominal value with a ± 15% tolerance at a specified number of turns from solid. Life should be specified as the

**Table 14-1.** Factor K for Calculating the Number of Revolutions a Power Spring Will Deliver.

$\frac{0.785 (D_c^2 - D_a^2)}{Lt}$	1.5	1.6	1.7	1.8	1.9	2.0
K	0.850	0.834	0.823	0.813	0.804	0.795

Calculate the expression and select the nearest value. Choose the appropriate K factor.



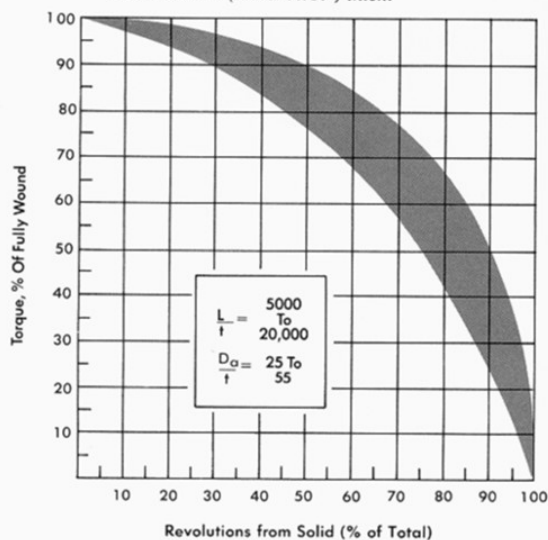
$S_{10}$  or cycle range in which 90% of the springs are expected to survive. Many power springs and pre-stressed power springs are supplied in a case with the inside end free. End positions are reference dimensions.

### 11.5 Power Spring Design Example

Assume a power spring is to deliver a minimum torque of 2.54 N · m (22.5 lbf-in) at eight full revolutions of the arbor. It is to work inside a 102 mm (4") diameter case. Width of stock is 19 mm (0.75"). What are the thickness  $t$  and active length  $L$ ?

1. It is best to design about 10% higher than minimum torque or  $2.54 + 10\% = 2.8 \text{ N} \cdot \text{m}$ .
2. Assume a load point at some middle position, say 66% of rotation to solid. Since 66% wound is the same position as 34% unwound, from Figure 14-4, it can be seen that the spring will produce 92% of full torque. The fully wound torque is  $2.80 \text{ N} \cdot \text{m} \div 0.92 = 3.04 \text{ N} \cdot \text{m}$ . Total number of revolutions will be  $8 \div 0.66 = 12.1$ . Because  $12.1 - 8 = 4.1$  is greater than 1, it is not necessary to add more material.

**Fig. 14-8. Typical Torque-Revolution Curve for Pre-stressed Power Springs. Made from Steel Strip 0.13-0.26 mm (0.005-0.010") thick.**



3. Using 19 mm width of stock, torque is  $3.04 \text{ N} \cdot \text{m} \cdot 10/19 = 1.60 \text{ N} \cdot \text{m}$ . Material thickness is 0.76 mm (Figure 14-5).

4. In Figure 14-7, the  $L/t$  ratio for 12.1 total revolutions is 5,500 and  $D_c/t$  ratio is 120; therefore, length  $L$  is  $0.76 \cdot 5500 = 4180 \text{ mm}$  and case diameter ( $D_c$ ) is  $0.76 \cdot 120 = 91.2 \text{ mm}$ . Assuming a nominal  $D_a/t$  ratio of 20, arbor diameter ( $D_a$ ) is  $0.76 \cdot 20 = 15.2 \text{ mm}$ . Rechecking the total available revolutions by:

$$\theta = \frac{\sqrt{2(D_c^2 + D_a^2)} - (D_c + D_a)}{2.55t}$$

$$\theta = \frac{\sqrt{2(91.2^2 + 15.2^2)} - (91.2 + 15.2)}{(2.55)(0.76)}$$

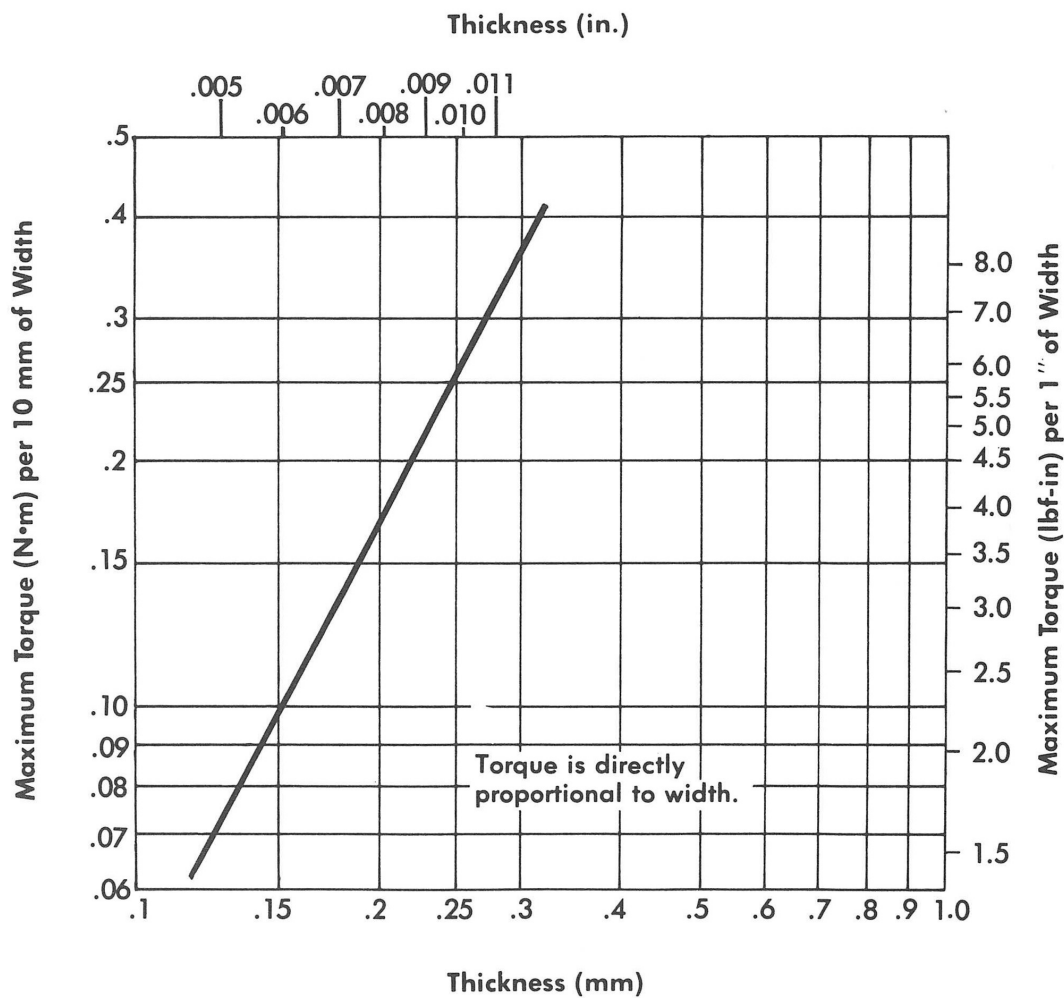
$$\theta = 12.56 \text{ revolutions}$$

This agrees closely with the original calculation of 12.1. Torque is directly proportional to width.

### 11.6 Final Design Specifications

- Thickness  $t$ :  $0.76 \pm 0.019 \text{ mm}$  ( $0.030 \pm 0.00075$ ")
- Width  $b$ :  $19 \pm 0.13 \text{ mm}$  ( $0.750 \pm 0.005$ ")
- Material: AISI 1095 carbon steel
- Length  $L$ : 4180 mm (164") Reference
- Case Diameter  $D_c$ : 91.2 mm (3.60")
- Shaft Diameter  $D_a$ : 15.2 mm (0.598")
- Minimum Torque at 8 Revolutions: 2.54 N · m (22.5 lbf-in)

**Fig. 14-9.** *Maximum Theoretical Torque at Solid for Pre-stressed Power Springs.*



## 12 CONSTANT FORCE SPRINGS

### 12.1 Introduction

A constant force spring is a roll of pre-stressed strip that exerts a nearly constant restraining force to resist uncoiling (Figures 15-1 and 15-2). The force is constant because the change in the radius of curvature is constant. This is true if the change in coil diameter due to buildup is disregarded.

Long extension capabilities, constant torque, and virtual absence of intercoil friction have led many designers to specify constant force springs in such applications as brush springs for motors, counterbalance springs for window sashes, and carriage return springs for typewriters. Constant force motor springs are used to drive mechanisms for timers, movie cameras, and cable retractors. Associated Spring RAYMOND has included many pre-engineered constant force extension springs in the SPEC product line. These springs are available for immediate delivery and are especially useful to designers and engineers for prototype applications.

### 12.2 Extension Type

Constant force extension springs are supplied as a coil of strip with a natural radius of curvature  $R_n$  (Figure 15-3). If such a spring is mounted on a drum, the drum diameter should be 10% to 20% larger than its natural diameter. One and one-half wraps should remain on the drum at maximum extension. The active portion of the material is approximately equal to 1.25 times the diameter  $D_D$ . Consequently, the spring does not reach its rated load until the extension is greater than  $1.25 D_D$  (Figure 15-1). Radius of curvature changes from  $R_n$  to infinity in the active portion

and requires a clearance. Recommended distance from drum center to the straight section is 0.8 times the diameter  $D_D$  (Figure 15-3). The strip becomes unstable at long extensions and should be guided to prevent twisting or kinking on recoil. Idler pulleys must be larger in diameter than the natural diameter and should never be used to cause back-bending against the natural radius of curvature (Figure 15-4). Idler rolls reduce the life of constant force springs. The maximum recommended bending stress levels based on strip with a No. 1 round edge are shown in Figure 15-7. Some typical methods for mounting extension type springs are illustrated in Figure 15-5.

*Fig. 15-1. Typical Constant Force Extension and Motor Springs.*

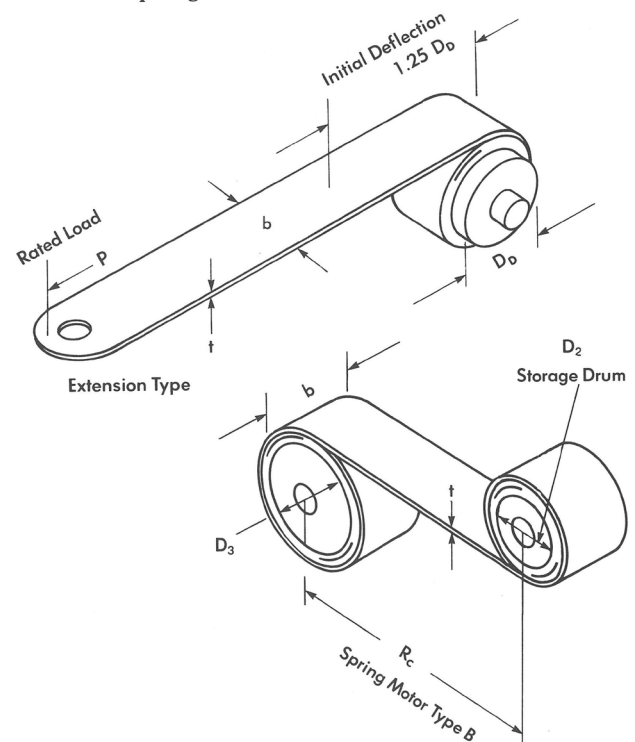


Fig. 15-2. Load Deflection Curves for Various Spring Configurations.

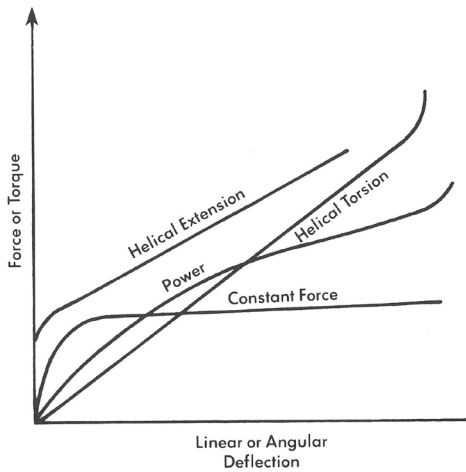
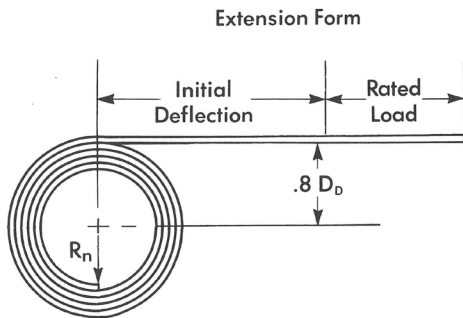


Fig. 15-3. Typical Constant Force Extension Spring.



The design equations for extension type springs are:

$$(15-1) \quad P = \frac{Ebt^3}{6.5D_a^2} \text{ for } N \leq 10$$

$$(15-2) \quad P = \frac{Ebt^3}{6.5D_1} \left( \frac{2}{D_n} - \frac{1}{D_1} \right) \text{ for } N > 10$$

$$(15-3) \quad S = \frac{Et}{D_n}$$

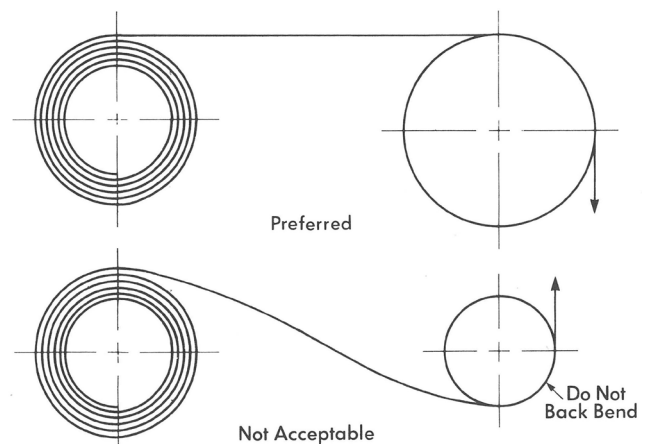
- $D_1$  = outside coil diameter
- $D_d$  = drum diameter
- $D_n$  = natural diameter
- $N$  = number of turns
- $L = 1.56 N (D_1 + D_d) = f + 5D_d$

- For typical designs, ratio  $b/t = 100$  and ratio  $D_d/D_n = 1.2$ .

### 12.3 Tolerances

Width tolerances are equal to the tolerances on the slit strip. Allowance must be made in the guides to accommodate edge camber inherent in strip. Load tolerances normally are held within  $\pm 10\%$ ; closer tolerances are available on request.

Fig. 15-4. Preferred Method for Using Idler Rolls with Constant Force Extension Springs.



### 12.4 Motor Type

Constant force spring motors are available in two configurations known as A and B motors (Figure 15-6). When a constant force spring is mounted on output and storage drums, the spring will tend to lower its potential energy by winding onto the storage drum. Torque can be applied or generated from the output drum. Constant torque should not be mistaken for constant speed. To achieve constant speed, the mechanism must be restrained by a governor or similar device.

The design equations for B motors are:



(15-5) 
$$M = \frac{Ebt^3D_3}{13} (1/D_n + 1/D_3)^2$$

(15-6) 
$$S = Et \left( \frac{1}{D_n} + \frac{1}{D_3} \right)$$

(15-7) 
$$L = \pi N (D_3 + Nt) + 10 D_3$$

(15-8) 
$$R_c = R_n \sqrt{4 + \frac{4R_3}{R_n} + \frac{R_n}{R_3} + \left(\frac{R_3}{R_n}\right)^2}$$

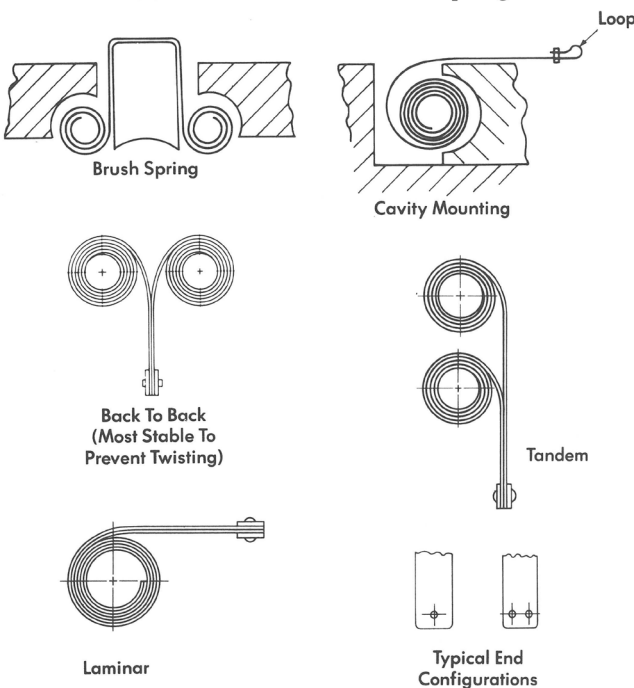
- $D_n$  = natural diameter
- $D_2$  = storage drum diameter
- $D_3$  = output drum diameter
- $R_3$  = output drum radius
- $R_c$  = center to center distance of drums

Note that for A motor springs,  $D_3$  is negative in the torque equation and drops out of the stress equation.

Typical values for the design parameters are:

$b/t = 100, D_n/t = 250, D_3/D_n = 2, D_3/D_2 = 1.6$

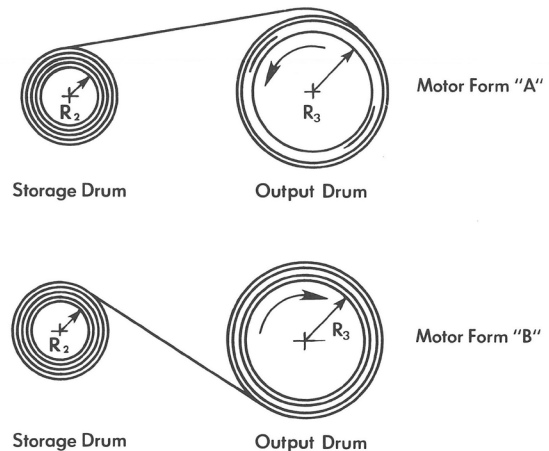
*Fig. 15-5. Typical Methods of Mounting and Attaching Constant Force Extension Springs.*



## 12.5 Choice of Stress Levels

Maximum recommended stress levels for constant force spring motors depend on desired life. Stress is estimated from Equation 15-6, which gives the theoretical fiber stress in a spring. Due to residual stresses, the actual stress experienced by a spring may be considerably less than indicated. However, in most cases, the Equation 15-6 gives a good stress approximation. Stress versus cycle life is presented in Figure 15-7. The term cycle refers to a stress cycle experienced by any portion of the spring. Moving an element of material through the active region constitutes a cycle for that element.

*Fig. 15-6. Typical Constant Force Spring Motor Configurations.*



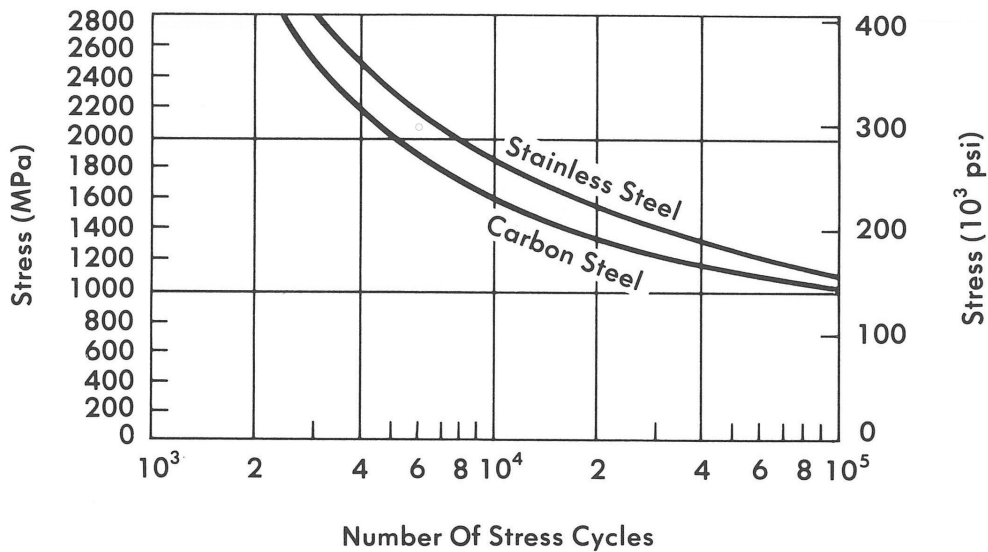
## 12.6 Tolerances

Width tolerances for constant force motor springs are dictated by strip tolerances, and diameter tolerances are determined by storage and output drum tolerances. Torque tolerances are  $\pm 10\%$ . In specifying constant force spring motors, only the space and functional requirements should have tolerances. Strip thickness and length should be considered reference dimensions.

12.7 How to Specify

The constant force spring specification checklist below is provided as a guide to all critical aspects of constant force springs.

**Fig. 15-7. Maximum Bending Stress versus Number of Stress Cycles for Constant Force Spring Motors. Based on No. 1 Round edge strip**



**CONSTANT FORCE SPRING SPECIFICATION CHECKLIST**

(Fill in required data only)

Type of Constant Force Spring:  
 Extension type \_\_\_\_\_, motor type \_\_\_\_\_

Material \_\_\_\_\_

**Working Conditions:**

Allowable space envelope for spring \_\_\_\_\_  
 Load \_\_\_\_\_ N(lbf) ± \_\_\_\_\_ N(lbf)  
 Torque \_\_\_\_\_ N · mm(lbf-in.) ± \_\_\_\_\_ N · mm(lbf-in.)  
 Extended length \_\_\_\_\_ mm(in.)  
 Required life \_\_\_\_\_ cycles  
 Number of revolutions required \_\_\_\_\_  
 Describe one cycle \_\_\_\_\_

**Special Information:**

Maximum operating temperature \_\_\_\_\_ °C(°F)  
 Operating equipment \_\_\_\_\_

**Reference Data:**

Thickness \_\_\_\_\_ mm(in.)  
 Width \_\_\_\_\_ mm(in.)  
 Length \_\_\_\_\_ mm(in.)  
 Material edge no. \_\_\_\_\_  
 Drum diameter (extension type) \_\_\_\_\_  
 Output drum diameter (motor type) \_\_\_\_\_ mm(in.)  
 Storage drum diameter (motor type) \_\_\_\_\_ mm(in.)  
 Natural diameter \_\_\_\_\_ mm(in.)  
 Center-to-center distance of drums \_\_\_\_\_ mm(in.)



## 13 SPIRAL SPRINGS

### 13.1 Introduction

This group includes hair springs and spiral torsion springs or brush springs. Spiral springs are characterized by the requirement that their coils do not touch during operation. Stress occurs in bending and, in most applications, deflections are limited to fewer than three revolutions.

### 13.2 Hair Springs

Hair springs are small spiral springs with space between coils. Hair springs are used in meters, instruments, and timing mechanisms, and are usually mounted on hubs or collets. The outer end is often clamped or hinged. If a hair spring is subject to deflections greater than two to three revolutions, the coils become unstable and buckle into a ball-like configuration called caging. In general, the width-to-thickness ratio  $b/t$  should be kept within a range of 3:1 to 15:1. The length to thickness ratio  $L/t$  should be in a 1000:1 to 3000:1 range. A typical design is one with a  $b/t = 10$  and  $L/t = 2000$ .

Deflection is given by:

(16-1)

$$\theta = \frac{6ML}{\pi bt^3 E}$$

(16-2)

$$L \simeq N_t \pi \left( \frac{D_o + D_i}{2} \right)$$

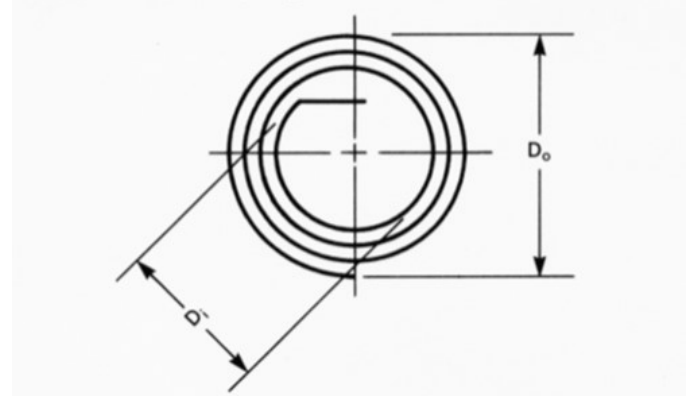
This equation applies when both ends are rigidly mounted. If the outer end is hinged, deflection for a given torque will be about 25% greater than calculated above. Spring rate is constant, provided the length of active material remains constant. Hysteresis is near zero.

Uncorrected stress is calculated from:

(16-3)

$$S = \frac{6M}{bt^2}$$

**Fig. 16-1.** Inside and Outside Diameters of a Typical Spiral Spring.



For fatigue applications, particularly for hair springs that contain sharp bends, a correction factor that accounts for curvature is used. Correction factors are expressed as:

Round wire:

(16-4)

$$K_{OD} = \frac{4C + 1}{4C + 4}$$

(16-5)

$$K_{ID} = \frac{4C - 1}{4C - 4}$$

Square wire:

(16-6)

$$K_{OD} = \frac{4C}{4C + 3}$$

(16-7)

$$K_{ID} = \frac{4C}{4C - 3}$$

### 13.3 Choice of Stress Level

Maximum corrected stresses for hair springs subject to cyclic service are presented in Table 16-1. These stresses are for a stress ratio of zero. For applications subject to stress reversal, maximum stress values should be reduced to 60% of stated values.

### 13.4 Tolerances

Hair springs are produced in a wide range of sizes. For small sizes, their tolerances depend on wire tolerances. Associated Spring engineers should be consulted for specific tolerance information.

### 13.5 Brush Springs

Spiral torsion springs are generally called brush springs because of their historic use in applying pressure to carbon brushes in motors and generators. Brush springs are seldom deflected more than 360° and generally have a length-to-thickness ratio  $L/t$  between 200 and 1000. The torque revolution curve is linear, provided that rotation is small, the coils do not touch each other, and the amount of active material remains constant. Design equations are the same as for hair springs.

Short brush springs with fewer than two turns will have a slightly higher rate than indicated in these equations.

Selecting an arbor diameter to determine space requirements for a brush spring is complex. The task is made easier by use of:

$$\theta \simeq \frac{\sqrt{D_a^2 + 1.27Lt} - D_a}{2t} - \frac{2L}{\pi(OD + D_a)} \quad (16-8)$$

$D_a$  = arbor diameter.

This approximation assumes that coils are spaced uniformly in the free position. Inactive material at points of attachment is not considered.

### 13.6 Choice of Stress Level

Maximum uncorrected stress levels for static applications are shown in Table 16-2. For fatigue applications, maximum corrected stress levels are presented as a function of life in Table 16-3. Correction factors are necessary to account for the stress concentration due to curvature (Equations 16-4, 16-5, 16-6, or 16-7).

*Table 16-1. Maximum Corrected Design Stresses for Hair Springs in Cyclic Applications.*

Material	Maximum Stress		Modulus E	
	MPa	psi	10 <sup>3</sup> MPa	10 <sup>6</sup> psi
Spring Steel	552	80,000	.207	30
18-8 Stainless	517	75,000	.193	28
Ni-Span C	552	80,000	.186	27
Beryllium Copper	414	60,000	.117	17
Phosphor-Bronze	414	60,000	.100	14.5

This information is based on ambient conditions. Stress ratio is 0. For a stress ratio of  $-1$ , these stress values must be multiplied by 0.6.

*Table 16-2. Maximum Uncorrected Design Stresses for Brush Springs in Static Applications.*

Material	Percent of Tensile Strength	
	Stress-Relieved	With Favorable Residual Stress
Carbon Steel	80	100
Nonferrous and Austenitic Stainless Steel	—	80

### 13.7 Tolerances

While the dimensional tolerances for brush springs vary with the application, load tolerances can be held within  $\pm 10\%$ .

### 13.8 How to Specify

The spiral spring specification checklist is provided as a guide to all critical aspects of spiral springs.

**Table 16-3. Maximum Corrected Design Stresses for Carbon Steel Brush Springs in Cyclic Applications.**

Fatigue Life (Cycles)	Percent of Tensile Strength
	Maximum Corrected Stress
10 <sup>4</sup>	80
10 <sup>5</sup>	62
10 <sup>6</sup>	60
10 <sup>7</sup>	58

This information is based on ambient conditions. Springs are free from burrs, notches and stress concentrations. Stress ratio is 0.

### 13.9 Brush Spring Design Example

A torque of 100 N · mm (0.885 lb-in) is required for 120° of revolution. The spring can be 6.5 mm (0.256") wide and will work on a 9.5 mm (0.374") diameter arbor. Material is carbon steel. Stress at 100 N · mm (0.885 lb-in) is 585 MPa (85,000 psi). Both ends are clamped and there are no sharp bends.

$$S = \frac{6M}{bt^2}, t = \sqrt{\frac{6M}{bS}} = \sqrt{\frac{6(100)}{6.5(585)}} = 0.397 \text{ mm}$$

$$\text{Then } L = \frac{\pi Ebt^3\theta}{6M}, \text{ when } \theta = \frac{120}{360} = 0.333 \text{ revolutions}$$

$$L = \frac{\pi(20.7 \times 10^4)(6.5)(0.397)^3(0.33)}{6(100)} = 146.8 \text{ mm}$$

$$\text{and } O.D. = \frac{2L}{\pi \left( \frac{\sqrt{D_a^2 + 1.27Lt} - D_a}{2t} - \theta \right)} - D_a$$

$$= \frac{2(146.8)}{\pi \left( \frac{\sqrt{(9.5)^2 + (1.27)(146.8)(0.397)} - 9.5}{2(0.397)} - 0.333 \right)} - 9.5$$

$$O.D. = 14.8 \text{ mm}$$

#### SPIRAL SPRING SPECIFICATION CHECKLIST

(Fill in required data only.)

Type of spiral spring:

Hair spring \_\_\_\_\_ brush spring \_\_\_\_\_

Material \_\_\_\_\_

Working Conditions:

To work over \_\_\_\_\_ mm(in.) diameter arbor/hub

To work in \_\_\_\_\_ mm(in.) diametral space

Torque \_\_\_\_\_ N·mm(lbf-in.) at \_\_\_\_\_ degrees of rotation

Torque \_\_\_\_\_ N·mm(lbf-in.) at \_\_\_\_\_ degrees of rotation

Required life \_\_\_\_\_

Describe one cycle \_\_\_\_\_

Special Information:

Maximum operating temperature \_\_\_\_\_ °C(°F)

Operating environment \_\_\_\_\_

Describe arbor/hub (sketch)

Describe inner and outer ends (sketch)

Reference Data:

Thickness \_\_\_\_\_ mm(in.)

Width \_\_\_\_\_ mm(in.)

Length \_\_\_\_\_ mm(in.)

Material edge no. \_\_\_\_\_



## 14 VOLUTE SPRINGS

### 14.1 Introduction

Volute springs are similar to conical compression springs. A volute spring consists of a relatively wide, thin strip of metal wound on the flat so that each turn or coil telescopes inside the preceding one. The material is stressed in torsion. Such a spring, when fully compressed, has a solid height equal to the strip width. Double volute springs are similar to two volute springs in series but must be made from blanked forms (adding to their cost).

Volute springs can store large amounts of energy in a small space. Other advantages include vibration damping provided by friction between the coils, extra absorption of impact energy, good lateral stability, and a nonlinear load deflection curve. The latter produces a rapidly increasing rate as coils bottom out in succession.

Disadvantages of volute springs are that intercoil friction causes galling and stress concentrations that may lead to early failure in cyclic applications. There is also a very nonuniform stress distribution within the spring, which tends to further reduce fatigue performance. If a nonlinear rate is undesirable, the load deflection curve can be made more linear by winding the larger coils with a greater helix angle, so all coils bottom out at the same time.

Volute springs are produced in a wide range of sizes. Small sizes are generally cold-wound from either pre-tempered or annealed carbon or spring temper stainless steel strip. Large sizes, often used in suspension and bumper systems, are usually hot-wound from carbon and low alloy steels.

### 14.2 Choice of Stress Level

Maximum operating stress levels are presented in Table 17-1 for static applications. Many large, hot-wound volute springs have set removed, while most cold-wound volute springs are supplied in stress-relieved condition.

*Table 17-1. Maximum Uncorrected Design Stresses for Volute Springs in Static Applications.*

Material	Percent of Tensile Strength	
	Stress-Relieved	With Set Removed
Carbon and Martensitic Stainless Steel	45	65
Nonferrous and Austenitic Stainless Steel	35	55

Maximum recommended stresses for cyclic service are given in Table 17-2. This applies only to volute springs that have no coil interference and are made from wire or strip with round edges.

### 14.3 Design Equations

Volute springs are usually mounted between flat plates so that all coils can bottom out. A bearing surface of at least three-quarters of a coil is required at each end. An approximate design technique uses methods developed for conical and rectangular wire springs.

The spring rate of an active element is determined from:

$$(17-1) \quad k = P/f = \frac{Gbt^3 K_2}{D^3 N_a}$$

where  $K_2$  is a constant shown in Figure 5-16. The rate for the entire spring is obtained from the rates of all active spring elements by:

(17-2)

$$k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \cdots + \frac{1}{k_n}}$$

(17-3)

$$S = \frac{PD_1K_F}{K_1bt^2}$$

$K_1$  is a constant shown in figure 5 – 16

where  $D_1$  is the mean diameter of the inside coil and  $K_F$  is given in Figure 5-17. For a spring with a constant helix angle, stress will be highest at this location. For variable pitch springs, stress is often highest at the outside coil. More detailed design methods are presented in Reference 3 and Reference 9. For critical applications, consult Associated Spring engineers for design assistance and tolerance recommendations.

**Table 17-2. Maximum Corrected Design Stresses for Carbon Steel Volute Springs in Cyclic Applications.**

Fatigue Life Cycles	Percent of Tensile Strength
	Maximum Design Stress
$10^4$	45
$10^5$	35
$10^6$	33
$10^7$	30

This information is based on ambient conditions. Strip has round edge or rolled wire. No intercoil friction. Stress ratio is 0.

## 15 WIRE FORMS

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### 15.1 Introduction

Wire forms are designed with a wide variety of shapes to perform many functions. Wire forms are often used as links to carry loads with a minimum deflection, electrical resistance units, or as rings and clips to hold parts in place. There are no general design equations for wire forms. The equations for torsion springs (Section 9) are often modified to determine the stresses and deflections due to bending, while the equations for compression springs (Section 5) are modified to determine the stresses and deflections due to twisting.

Wire forms are made from both high-strength spring materials and annealed materials. Sharp bends are not only difficult to form but cause stress concentrations, and they should be avoided whenever possible. Most wire forms are manufactured in one operation on automatic equipment; therefore, tooling is required.

### 15.2 How to Specify

An engineering drawing is the best way to specify the wire form. Consultation with Associated Spring engineers early in the design stage of wire forms will often result in considerable cost savings. Also, minor changes in part design can sometimes facilitate manufacture on automatic machinery, as well as reduce tool costs.

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## TRADEMARKS

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Associated Spring

SPRING DESIGN

QUICK REFERENCE GUIDE

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## GLOSSARY OF SPRING TERMINOLOGY

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### Active coils

Those coils that are free to deflect under load.

### Angular relationship of ends

Relative position of hooks or loops of an extension spring (or ends of a torsion spring) to each other.

### Baking

Heating of electroplated springs to relieve hydrogen embrittlement.

### Block

See Solid height.

### Buckling

Bowing or lateral displacement of a compression spring. This effect is related to slenderness ratio  $L/D$ .

### Close wound

Adjacent coils are touching.

### Closed and ground ends

Same as Closed ends, except the first and last coils are ground to provide a flat bearing surface.

### Closed ends

Compression spring ends with coil pitch angle reduced so they are square with the spring axis and touch the adjacent coils.

### Closed length

See Solid height.

### Coils per inch

See Pitch.

### Deflection

Motion imparted to a spring by application or removal of an external load.

### Elastic limit

Maximum stress to which a material may be subjected without permanent set.

### Endurance limit

Maximum stress, at a given stress ratio, at which material will operate in a given environment for a stated number of cycles without failure.

### Fixture tempering

Restraining parts during tempering to improve dimensional control.

### Free angle

Angular relationship between arms of a helical torsion spring that is not under load.

### Free length

Overall length of a spring that is not under load.

### Gradient

See Rate.

### Heat setting

A process to pre-relax a spring in order to improve stress relaxation resistance in service.

### Helical springs

Springs made of bar stock or wire coiled into a helical form. This category includes compression, extension, and torsion springs.

**Hooks**

Open loops or ends of extension springs.

**Hysteresis**

Mechanical energy loss occurring during loading and unloading of a spring within the elastic range. It is illustrated by the area between load-deflection curves.

**Initial tension**

A force that tends to keep coils of a close-wound extension spring closed and that must be overcome before the coils start to open.

**Loops**

Formed ends with minimal gaps at the ends of extension springs.

**Mean diameter**

The average diameter of the mass of spring material, equal to one-half the sum of the outside and inside diameters. In a helical spring, this is the equivalent to the outside diameter minus one wire diameter.

**Modulus in shear or torsion (modulus of rigidity G)**

Coefficient of stiffness used for compression and extension springs.

**Modulus in tension or bending (Young's Modulus E)**

Coefficient of stiffness used for torsion or flat springs.

**Moment**

A product of the distance from the spring axis to the point of load application, and the force component normal to the distance line.

**Natural frequency**

Lowest inherent rate of free vibration of a spring vibrating between its own ends.

**Patenting**

The process of heating carbon steel above its critical temperature and cooling at a controlled rate to achieve a fine pearlitic microstructure.

**Pitch**

Distance from center to center of wire in adjacent coils in an open-wound spring.

**Plain ends**

End coils of a helical spring having a constant pitch and ends not squared.

**Plain ends, ground**

Same as Plain ends, except wire ends are ground square with the axis.

**Rate**

Spring gradient or change in load per unit of deflection.

**Residual stress**

Stress mechanically induced by such means as set removal, shot-peening, cold working, or forming. It may or may not be beneficial, depending on the spring application.

**Set**

Permanent change of length, height, or position after a spring is stressed beyond material's elastic limit.

**Set point**

Stress at which some arbitrarily chosen amount of set (usually 2%) occurs. Set percentage is the set divided by the deflection that produced it.

**Set removal**

An operation that causes a permanent loss of length or height due to spring deflection.

**Shot-peening**

Blasting the surfaces of spring material with steel or glass pellets to induce compressive stresses that improve fatigue life.

**Slenderness ratio**

Ratio of spring length to mean diameter  $L/D$  in helical springs.

**Solid height**

Length of a compression spring when deflected under sufficient load to bring all adjacent coils into contact.

**Spiral springs**

Springs formed from flat strip or wire wound in the form of a spiral, loaded by torque about an axis normal to the plane of the spiral.

**Spring index**

Ratio of mean diameter to wire diameter.

**Squared and ground ends**

See Closed and ground ends.

**Squared ends**

See Closed ends.

**Squareness**

Angular deviation, between the axis of a compression spring in a free state and a line normal to the end planes.

**Stress range**

Difference in operating stresses at minimum and maximum loads.

**Stress ratio**

Minimum stress divided by maximum stress.

**Stress relief**

A low temperature heat treatment given springs to relieve residual stresses produced by prior cold forming.

**Torque**

See Moment.

**Total number of coils**

The sum of the number of active and inactive coils in a spring body.

## CONVERSION FACTORS

### METRIC AND ENGLISH UNITS

#### Conversion Factors, Metric and English Units

	To Convert	Multiply By	To Obtain
Area	in <sup>2</sup>	645.2	mm <sup>2</sup>
Density	lb/in <sup>3</sup>	27.68	g/cm <sup>3</sup>
Force	lbf	4.448	N
	ozf	0.278	N
	lbf	0.4536	kgf
Frequency	c/s	1.000	Hz
Length	in.	25.4	mm
	ft.	304.8	mm
Mass	lb	0.4536	kg
Moduli (E and G)	psi	0.006895	MPa
Pressure	psi	6.895	kPa
Spring Rate	lb/in.	0.1751	N/mm
	lb/in.	0.01785	kg/mm
Stress	psi	0.006895	MPa
Torque	lbf-in.	112.9	N·mm
	lbf-ft	1355	N·mm
	ozf-in.	7.062	N·mm
Temperature	°F	$= (°F - 32) \div 1.8$	°C
	°C	$= 1.8°C + 32°$	°F
Tensile Strength	psi	0.006895	MPa
Volume	in <sup>3</sup>	16,387	mm <sup>3</sup>
Work	ft-lbf	1.365	J

## CONVERSIONS FOR CARBON STEEL

BETWEEN HARDNESS SCALES AND TENSILE STRENGTHS, APPROXIMATE

C	A	15N	30N	45N	10 kg DPH	Vickers	500 g Knoop	3000 kg Brinell	Tensile Strength, Approximate		B	15T	30T	45T
									MPa	10 <sup>3</sup> psi				
66	84.5	92.5	83	73	854	860	870							
64	83.5	92	81	71	789	800	822							
62	82.5	91	79	69	739	740	776							
60	81	90	77.5	66.5	695	697	732	614						
58	80	89.5	75.5	64	655	655	690	587						
56	79	88.5	74	62	617	615	650	560						
54	78	87.5	72	59.5	580	579	612	534	2020	293				
52	77	86.5	70.5	57.5	545	547	576	509	1889	274				
50	76	85.5	68.5	55	513	516	542	484	1765	256				
48	74.5	84.5	66.5	52.5	485	482	510	460	1641	238				
46	73.5	83.5	65	50	458	458	480	437	1538	223				
44	72.5	82.5	63	48	435	435	452	415	1441	209				
42	71.5	81.5	61.5	45.5	413	410	426	393	1331	193				
40	70.5	80.5	59.5	43	393	396	402	372	1248	181				
38	69.5	79.5	57.5	41	373	372	380	352	1179	171				
36	68.5	78.5	56	38.5	353	350	360	332	1110	161				
34	67.5	77	54	36	334	336	342	313	1048	152				
32	66.5	76	52	33.5	317	319	326	297	1007	146				
30	65.5	75	50.5	31.5	301	301	311	283	952	138				
28	64.5	74	48.5	29	285	285	297	270	903	131				
26	63.5	72.5	47	26.5	271	272	284	260	862	125				
24	62.5	71.5	45	24	257	260	272	250	827	120				
22	61.5	70.5	43	22	244	247	256	240	786	114	100	93	82	72
20	60.5	69.5	41.5	19.5	228	241	241	230	758	110	98	92.5	81	70
	59				216		231	215	703	102	96	92	80	68
	57.5				205		221	205	676	98	94	91.5	78.5	66
	56.5				195		211	198	648	94	92	90.5	77.5	64.5
	55.5				185		201	185	614	89	90	90	76	62.5
	54				176		192	176	586	85	88	89.5	75	60.5
	53				169		184	169	558	81	86	88.5	74	58.5
	52				162		176	156	517	75	84	88	73	57
	50.5				156		170	151	503	73	82	87.5	71.5	55
	49.5				150		164	150	496	72	80	86.5	70	53
	48.5				144		158		483	70	78	86	69	51
	47				139		152		469	68	76	85.5	67.5	49

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